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Assessment of two methods for the accurate prediction of transverse stress distributions in laminates

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ABSTRACT

A major challenge for crash failure analysis of laminates is to find a modelling approach which is both sufficiently accurate and computationally efficient. We suggest to adopt a traditional single-layer shell formulation due to its cost effectiveness. In this contribution, we have therefore investigated the potential of two different concepts for obtaining better prediction of the through-the-thickness distribution of the transverse stresses; a crucial issue since the accuracy for a single-layer approach in this respect is normally low. The first concept is a multiscale approach in which the macroscopic shell model is concurrently coupled to a mesoscopic 3D element representation of the heterogeneous material structure on the laminate level. The second concept is a stress recovery method based on integration of the 3D equilibrium equations, with additional smoothing of the in-plane stresses.

The main conclusion drawn from the investigations is that, the adopted multiscale concept, although similar to what has been previously reported in the literature, is not a suitable approach to increase the level of accuracy of the predicted transverse stress distributions. However, we conclude that the proposed stress recovery method very well captures the through-the-thickness stress variations in our presented examples.

1. Introduction

Numerical finite element (FE) tools for the accurate prediction of the crash response of vehicle structures in fibre reinforced plastics (FRP) are crucial for structural composites to have a widespread use in future cars [1]. Traditionally, FE crash simulations are performed using shell element models, which are well suited to model the thin-walled metal structures in automotive bodies while at the same time being computationally cost effective compared to continuum solid (3D) element models. However, a known drawback of traditional shell element formulations is low accuracy of the through-the-thickness variation of the transverse stress components [2]. Thus, to be able to have a good level of predictability when simulating progressive crash failure in FRPs (*e.g.* to capture delamination, driven by high transverse stresses), better suited types of FE-models, than those traditionally used to model metals, need to be adopted.

A major challenge for crash failure analysis of laminates is thereby to find a modelling approach which is both sufficiently accurate and computationally efficient – a challenge addressed in this paper. Seeking a good compromise, we note that methods for simulating the structural behaviour of a laminated structure in a FE-framework can, following Reddy [2], generally be divided into two categories. Either layer-wise models (LWM), where each ply (or ply interface) of the laminate is represented by separate degrees of freedom (DoF),[1] or equivalent single-layer models (ESLM) where one layer of shell elements is used to represent the entire laminate. In the review by Carrera [3], it was concluded that the accuracy of the transverse stresses in LWM were superior compared to ESLM and that mixed formulations, where *e.g.* the transverse stress components can be regarded as unknown DoF, showed superior accuracy compared to traditional pure displacement type ones. Please refer also the review on Reissner Mixed Variational Theorem (RMVT) by Carrera [4], where a unified formulation is introduced, and the general review on modelling of FRP laminates by Kreja [5]. On the other hand, ESLM are more computationally efficient compared to LWM and if the accuracy of the transverse stresses can be improved, they will be highly competitive.

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[1] This category includes the 3D element models and the stacked shell element models, where each ply is modelled with at least one layer of elements in the thickness direction.

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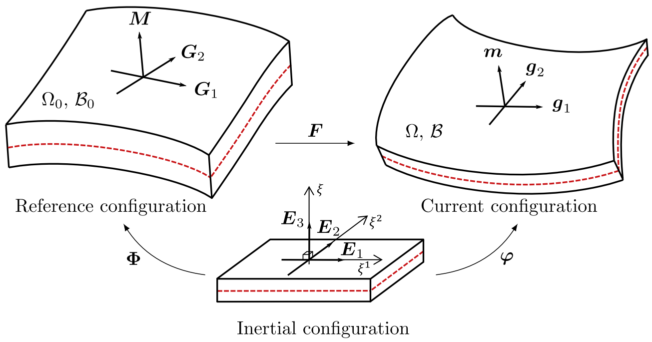

Fig. 1. Mappings of shell model defining undeformed and deformed shell configurations relative to inertial Cartesian frame.

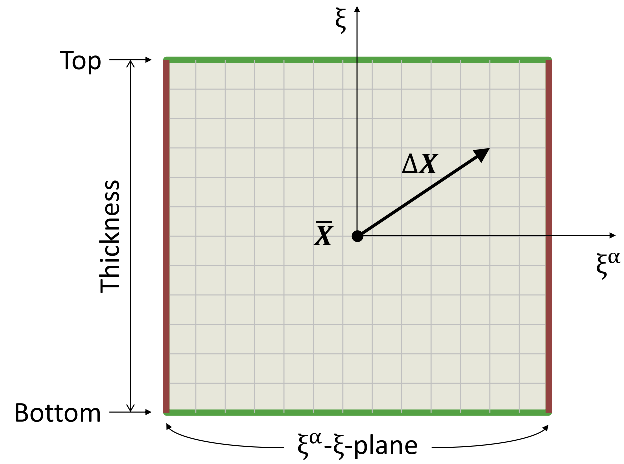

Fig. 2. Sketch of midsection through the $12 \times 13 \times 13$ (hexahedra) element RVE showing the expansion point $\bar{\boldsymbol{X}}$ and the placement vector $\Delta\boldsymbol{X}$ in the reference configuration.

The subject of improving the transverse stresses in ESLM have been addressed by many authors over the years. Recently the work by Carrera and co-workers was extended to construct an advanced locking-free finite element based on the RMVT formulation [6]. Besides adopting such *a priori* assumptions like mixed formulations or Zig-Zag in the shell formulation (see *e.g.* the historical review by Carrera [7]), *a posteriori* methods can be adopted [3].

All in all, and despite the conclusions made by Carrera [3] we suggest to adopt an ESLM shell formulation due to its cost effectiveness compared to LWM. The main argument for this is that LWM would for many real laminates require a too large amount of DoF which would be directly inappropriate for crash or other large scale analyses.

Therefore, the main goal of this work has been to establish a robust modelling method, which benefits from the cost efficiency of an ESLM, but at the same time yields accurate predictions of the transverse stresses. As an initial step, we will in this paper focus on the transverse shear stresses.

For this purpose, two methods have been implemented and assessed in terms of accuracy of the transverse shear stress distribution. First, we have investigated the potential of using a multiscale approach as a possible remedy to the problem of using ESLM stated above. A long term idea of adopting such an approach is to enable a model-adaptivity procedure, *cf. e.g.* Oden and Vemaganti [8], where initially the model is build up as an ESLM. Based on some measure, either a model error estimator or a failure initiation criterion, a transition to a coupled multiscale approach could be made locally in critical areas. In particular, we have adopted the multiscale concept introduced by Larsson and Landervik [9] for simulating deformations of thin-walled porous structures by coupling the macroscopic shell model to a mesoscopic 3D element representation of the heterogeneous material structure. Due to their promising results, our intention in this paper has been to investigate if a similar procedure can be adopted for simulating progressive failure in a laminated FRP plate. The main conclusion drawn from the investigations presented in this paper is however that, the concept proposed in [9] is not a suitable approach to increase the level of accuracy of the predicted transverse stress distributions.

As an alternative method, we have identified a suitable, and seemingly robust, post-processing procedure which allows accurate predictions of the transverse stress distribution to be made. This procedure is based on a nodal recovery of the in-plane stress components, averaged over neighbouring elements, followed by an integration of the transverse stress components using the 3D equilibrium equations. These recovered stresses can then be used in an initiation criterion for interlaminar crack nucleation, after which the delamination can be modelled using *e.g.* an appropriate cohesive formulation.

1.1. Outline of paper

First we set out to describe the adopted shell formulation in Section 2 together with a motivation of this particular choice. In Section 3, we continue by presenting the kinematics and choice of boundary conditions for the mesoscopic model necessary as part of the multiscale approach. Then in Section 4, our post-processing procedure is described and in Section 5 numerical examples are presented, which compare results from both the multiscale and the post-processing method. Detailed studies illustrating the effect from choosing different boundary conditions and RVE sizes are presented for an isotropic and a laminated cantilever beam. Finally, conclusions and discussions thereon are presented in Section 6.

2. Shell formulation

In this section, we will describe the underlying shell element formulation adopted in the current work, which is identical to what was proposed by Larsson and Landervik [9]. Thus, we adopt a solid-like ESLM shell formulation based on first-order shear deformation theory (FSDT) with a second-order expansion of the deformed configuration in the normal direction leading to a 7-parameter displacement formulation. The main ingredients of this formulation is repeated below, where in the subsequent text we let Latin letters denote the range from 1 to 3 and Greek letters denote the range from 1 to 2.

2.1. Reference and current shell geometry in terms of convected coordinates

As a staring point, the undeformed (reference) configuration $\mathcal{B}_0$ of the shell is considered parametrised in terms of convected coordinates $\boldsymbol{\xi}$ as

$$\mathcal{B}_0 = \left\{ \boldsymbol{X} := \boldsymbol{\Phi}(\boldsymbol{\xi}) = \boldsymbol{\Phi}_0(\boldsymbol{\xi}_0) + \xi \boldsymbol{M}(\boldsymbol{\xi}_0) : \ \boldsymbol{\xi}_0 \in A, \ \xi \in \frac{h_0}{2}[-1, 1] \right\} \quad (1)$$

where we introduced the compact notations $\boldsymbol{\xi} = (\xi^1, \xi^2, \xi^3 = \xi)$ and $\boldsymbol{\xi}_0 = (\xi^1, \xi^2)$ and where the mapping $\boldsymbol{\Phi}$ maps the inertial Cartesian frame into the undeformed configuration as shown in Fig. 1. In Eq. (1), the mapping $\boldsymbol{\Phi}$ is defined by the midsurface placement $\boldsymbol{\Phi}_0$ and the outward unit normal director field $\boldsymbol{M}$. The coordinate ξ is associated with the normal director field, h_0 is the initial thickness of the shell and A is the midsurface area. Associated with the