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A comparative study of metamodeling methods for the design optimization of variable stiffness composites

Mahdi Arian Nik, Kazem Fayazbakhsh, Damiano Pasini *, Larry Lessard

Department of Mechanical Engineering, McGill University, Macdonald Engineering Building, 817 Sherbrooke West, Montreal, QC, H3A 0C3, Canada

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ABSTRACT

Automated fiber placement is a manufacturing technology that enables to build composite laminates with curvilinear fibers. To determine their optimum mechanical properties, finite element analysis is commonly used as a solver within an optimization framework. The analysis of laminates with curvilinear fibers coupled with the fiber path optimization requires a large number of function evaluations, each time-consuming. To reduce the time for analysis and thus for optimization, a metamodel is often proposed. This work examines a set of metamodeling techniques for the design optimization of composite laminates with variable stiffness. Three case studies are considered. The first two pertain to the fiber path design of a plate under uniform compression. The third concerns the optimization of a composite cylinder under pure bending. Four metamodeling methods, namely Polynomial Regression, Radial Basis Functions, Kriging and Support Vector Regression, are tested, and their performance is compared. Accuracy, robustness, and suitability for integration within an optimization framework are the appraisal criteria. The results show that the most accurate and robust models in exploring the design space are Kriging and Radial Basis Functions. The suitability of Kriging is the highest for a low number of design variables, whereas the best choice for a high number of variables is Radial Basis Functions.

1. Introduction

Automated fiber placement (AFP) is a technology capable of placing fibers along a curvilinear path, thereby resulting in a variable stiffness laminate. The structural benefits of variable stiffness laminates are achieved by tailoring the material properties in directions that are more favorable to carry loads within the laminates. To fully exploit the advantages of a variable stiffness design, it is often appropriate to systematically formulate the design problem within an optimization framework. The objective functions to optimize might be one or more mechanical properties, such as buckling and in-plane stiffness. Since the fiber orientation continuously changes within the laminate of a variable stiffness design, the evaluation of the structural properties via finite element simulation is often very time-consuming [1,2]. Furthermore, the optimization process might require thousands of function evaluations to locate a near optimal solution, a requirement that makes the process computationally expensive. To alleviate this problem, one may resort to an approximation concept, also called a metamodel [3,4]. Significantly cheaper to evaluate, the metamodel is substituted and used in place of a high fidelity finite element simulation. As a result, the metamodel can significantly reduce the time required to run the optimization.

In the literature, there are several successful applications of metamodeling techniques in the optimization of traditional composite laminates with straight fibers. For example, Radial Basis Functions [5], second order polynomials [6], and Neural Networks [7] were shown to be effective in reducing the time to find the maximum buckling load of a composite stiffened panel. Liu et al. [8] used a cubic response surface combined with a two-level optimization technique to maximize the buckling load of a composite wing. Lee and Lin [9,10] used trigonometric functions as the base functions to build a metamodel for the stacking sequence optimization of a composite propeller. Integrated into a genetic algorithm (GA), the metamodel demonstrated benefits by reducing the number of GA iterations. Kalnins et al. [11] compared the performance of Radial Basis Functions, multivariate adaptive regression splines, and polynomials, to optimize the post-buckling of a damaged composite stiffened structure. They concluded that the methods under investigation have cross-validation error lower than 10%; thus, they can be efficiently integrated into an optimization framework. In another attempt, Lanzi and Giavotto [12] compared the performance of Radial Basis Functions, Neural Networks, and Kriging metamodels in a multi-objective optimization problem for maximum post-buckling load and minimum weight of a composite

* Corresponding author. Tel.: +1 514 398 6295; fax: +1 514 398 7365.
E-mail address: damiano.pasini@mcgill.ca (D. Pasini).

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stiffened panel. The methods were found to yield similar results and none of them was identified as being significantly superior.

While there is a considerable amount of existing research on the use of metamodels for constant stiffness composite design, only a few attempts look at their application in variable stiffness design. Among those worthy to mention are the following: the optimization of a variable stiffness laminate in vibration [13], the buckling load of a variable stiffness composite cylinder [14], and the simultaneous optimization of the buckling load and in-plane stiffness of a variable stiffness laminate ignoring the presence of defects, i.e. gaps and overlaps [2]. Recently, Arian Nik et al. [32] used the defect layer method [15] and a Kriging metamodel to simultaneously maximize the buckling load and in-plane stiffness of a variable stiffness laminate with embedded defects. While these works demonstrate the potential of a given metamodel in reducing the computational burden of the optimization process, they are just a first attempt. No recommendation about metamodel selection for variable stiffness composites exists. Furthermore, metamodel performance is problem dependent and the best metamodel is unknown at the outset [16].

This work presents a comparative study on the application of the most widely used metamodeling methods – Polynomial Regression, Radial Basis Functions, Kriging, and Support Vector Regression, for the optimization of variable stiffness composite. The goal is to offer insight into the selection of the most appropriate metamodel for the optimization of laminated composites with varying fiber angles. We examine three case studies: the buckling load and in-plane stiffness of a variable stiffness composite plate under uniform compression for two layup designs, and a variable stiffness composite cylinder under pure bending. The advantages and disadvantages of the metamodels are then investigated using the following criteria:

- *Accuracy*: the degree of closeness of a metamodel prediction to that quantity of the true function over the design range of interest. Multiple metrics, namely *R*-square, relative average absolute error, and relative maximum absolute error are used to assess the metamodels' accuracy.
- *Robustness*: the capability of a metamodel to persistently achieve high accuracy for dissimilar problems. In this work, the robustness of a metamodel method is measured by evaluating its average accuracy for the entire set of test problems.
- *Suitability*: the degree of the effectiveness of integrating a metamodel into an evolutionary optimization algorithm. To measure this criterion, the performance of metamodel-assisted optimization algorithms in the actual improvement of the solution is compared via a series of numerical experiments on the case studies.

The remainder of this work is organized as follows: the data sampling method and the different size of the sample data to investigate its effect on the metamodel accuracy are explained in Section 2. Section 3 gives a background on metamodel construction techniques and their characteristics. The metrics to evaluate the local and global metamodel accuracy are discussed in Section 4. Test problems for variable stiffness composite that can be manufacturable via AFP are then described in Section 5. Finally, the metamodels under investigation are assessed and recommendations are presented in Section 6.

2. Data sampling

Data sampling, referred to as design of experiments (DOE), is the first step in the construction of a metamodel. The selection of the sample points and the size of the sample have a significant effect on the metamodel accuracy.

Sacks et al. [17] stated that sample points for simulated experiments should be chosen to fill the design space rather than to concentrate on the boundaries of the design space. The reason is that computer experiments are deterministic and thus involve systematic errors, whereas physical experiments involve random errors. Following this observation, in this work a Latin Hypercube method is used to generate training data that are space filling. In addition, to average out the dependency of the metamodels accuracy on the sampling method, we use five DOEs to construct each metamodel.

Besides the sampling method, the sample size also has an influence on metamodel accuracy. To investigate the metamodel accuracy with respect to the sample size, small and large sample sizes are examined as suggested by Jin et al. [18]. Table 1 shows the sample sizes and the number of confirmation data points used to measure metamodel accuracy with respect to the sample size.

3. Metamodeling techniques

As mentioned in the introduction, there is a variety of techniques that can be used to construct a metamodel. This section gives a background on the most common methods: Polynomial Regression (PR); Radial Basis Functions (RBF); Kriging (KRG); and Support Vector Regression (SVR).

3.1. Polynomial Regression (PR)

A second-order polynomial can be expressed as

$$\tilde{y}(x) = \beta_0 + \sum_{i=1}^{n}\beta_i x_i + \sum_{i=1}^{n}\beta_{ii}x_i^2 + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\beta_{ij}x_i x_j \tag{1}$$

where β_0, β_i, β_{ii} and β_{ij} ($i = 1,\ldots,n$; $j = 1,\ldots,n$) are the regression coefficients, $x_i(i = 1,\ldots,n)$ are the design variables, and $\tilde{y}$ denotes the approximate value for the objective function. The coefficients of the metamodel are evaluated by fitting the model to the training data using the least squares method [19]. The second order PR has a smoothing capability, a feature that ensures fast convergence for noisy functions and thus is suitable for integration in an optimization framework. Yet, this characteristic can bring inaccuracy if there is need to surrogate highly non-linear functions [18]. Obviously, a higher order polynomial can be used to construct a more accurate metamodel; nevertheless, instabilities may arise and also a large number of training data is required to fit such a high order polynomial [20].

3.2. Radial Basis Functions (RBF)

The RBF method uses a combination of basis functions expressed in terms of the Euclidean distance between sample data points to construct a metamodel [21]. The RBF model can be written as

$$\tilde{y}(x) = \sum_{i=1}^{n} w_i \psi(\|x - x_i\|) \tag{2}$$

where x_i ($i = 1,\ldots,n$) are the design variables, ψ is the basis function and w_i ($i = 1,\ldots,n$) are the basis function weights evaluated by fitting the model to the training data, $||\cdot||$ denotes the Euclidean distance between two sample data points, and $\tilde{y}$ is the approximate value of the objective function [4]. The basis function weights, w_i, can be calculated by enforcing the interpolation condition in Eq. (2). This results in a linear system of equations

$$\mathbf{y} = \boldsymbol{\psi}\mathbf{w} \tag{3}$$

where $\mathbf{y}$ is the vector of function values at training data, $\mathbf{w}$ is the vector of basis function weights, and $\boldsymbol{\psi}$ is a matrix, also known as Gramian matrix of design variable values defined by

$$\boldsymbol{\psi} = \begin{vmatrix} \psi(x_1,x_1) & \psi(x_1,x_2) & \cdots & \psi(x_1,x_{nc}) \\ \psi(x_2,x_1) & \psi(x_2,x_2) & \cdots & \psi(x_2,x_{nc}) \\ \vdots & \vdots & \ddots & \vdots \\ \psi(x_{nc},x_1) & \psi(x_{nc},x_2) & \cdots & \psi(x_{nc},x_{nc}) \end{vmatrix} \tag{4}$$