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Maximum entropy method for probabilistic bearing strength prediction of pin joints in composite laminates

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ABSTRACT

The mechanical properties of composite materials exhibit a much wide range of scatter than that of metallic materials, even under the same loading condition. A general computational framework for probabilistic strength prediction of pin joints in composite laminates using maximum entropy method is proposed and developed in this study. The deterministic progressive failure analysis of composite pinned joint is based on a two-dimensional ABAQUS model, which takes into account large deformation theory and the non-linear shear behavior of composite material. Uncertainties arising in the mechanical properties and ultimate strengths of one lamina are modeled as random variables and considered in the process of probabilistic strength prediction. Point estimate method is used to estimate the first four statistical moments of bearing strength of composite pinned joint. Incorporating all those statistical moments, a maximum entropy method is used to approximate its probability distribution to avoid the pitfall encountered in Edgeworth expansions, which is a popular tool to approximate a probability distribution. An example of T300/QY8911 composite plate is given to demonstrate the proposed computational framework. Comparisons between the proposed framework and Edgeworth expansions based method are also performed to illustrate the advantage of the proposed framework.

1. Introduction

Composite materials have been increasingly used in not only aeronautical applications but also other engineering applications due to their many unique advantages, such as higher stiffness and strength to weight ratios, corrosion resistance, long fatigue life, and designability, compared with conventional metal materials. The use of mechanical joints is often required between composite structures with other structural elements to transfer loads and provide the flexibility of maintenance. However, the anisotropic nature and brittleness of most composite materials causes composite joints to exhibit complex stress field and multiple failure modes. Over the last two decades, the engineering research community has realized the importance of composite joint. As a result, many research studies have been focusing on this topic. Two comprehensive review articles of various historic periods can be found in [1,2].

In many practical applications, one of the most important and challenging problem is to predict and/or assess the strength of composite joint. Due to difficulties in analytical computing stress states in the vicinity of the hole, finite element (FE) method has been gained increasing popularity. Based on FE method, various progressive failure models were developed to predict the strength of composite joints [1–9]. Instead of creating their own FE code, many researchers have been endeavoring to utilize commercial FE software to build up FE model, such as ABAQUS [4,5], ANASYS [8,9]. The FE model developed by Dano et al. [4,5], which is based on ABAQUS' example [10], considers the influences of contact between pin and hole, large deformation theory, progressive damage and a non-linear shear stress–strain relationship. It was also shown that good agreements with experimental data were observed, although FE model is a two-dimensional (2D) one, which cannot generally take into account stacking sequence, clamping torque and delamination effects. Since three-dimensional (3D) FE model may result in more developed efforts and higher computational costs, this 2D FE model is employed for the bearing strength prediction in this study.

It is well-known that complicated manufacturing processes and the naturally anisotropic material behavior cause the phenomenon that the mechanical properties of composite materials usually display more inherit scatter than that of metallic materials. Furthermore, the process of drilling holes may induce defects in the laminated composite plates [11–13], which has a significant impact on the strength of composite joints because of the high stress concentrations near the pin-loaded holes. These uncertainties are usually bypasses by the use of the well-known "safety factors" in engineering community, without direct connection with

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themselves. As a matter of fact, the safety factors are calibrated by the average situations, and often yields an unreliable result including either conservative or non-conservative compared with the experimental and/or operational data. Thus, the need considering uncertainties in composite joints has led to the introduction of probabilistic methods to estimate the strength of pin-loaded composite joints [14,15], where all uncertainties are modeled probability by random variables. To qualify the influences of probabilistic uncertainties on the strength of composite pinned joint, simulation-based methods, for example Monte Carlo method, seem much straightforward. However, their higher computational cost hinders the application of them when structural FE analysis for pin joint is time-consuming or the required reliability level is very high. First-order reliability method is another kind of popularly probabilistic method. The linear approximation, however, may not be suitable for some cases, such as the strong non-linearity induced from FE model or non-Gaussian distribution transformations. In addition, this method is required to repeat for a range of thresholds to extract a cumulative distribution function (CDF) of composite joint strength. This increases the computational cost. Considered the uncertainties in the mechanical properties and ultimate strengths of unidirectional lamina, Li et al. [14] developed an efficient method for predicting probabilistic bearing strength of composite pinned joint using point estimate method (PEM) [16–18] and Edgeworth expansions [18]. However, a major disadvantage of this method is that Edgeword expansions was observed with failing to fit the CDF of composite joint strength in some cases when the coefficients of variation (cov) for some mechanical properties are relatively large. Since these estimated CDFs do not satisfy their mathematical definition, there is no any physical or mathematical meaning in them. Nakayama et al. [15] applied response surface method to approximate the effect induced by the variation of hole diameter on the pin joint strength. Monte Carlo method was then employed to obtain probability density function (PDF) of bearing strength of pin joint on the basis of response surface approximation. However, only one uncertain parameter, i.e. hole diameter, was considered and studied in their work.

Regarding the disadvantages of the existing methods, attention of this paper is to develop a more general computational framework for probabilistic bearing strength prediction of composite pinned joint. In this new framework, the probability distribution of bearing strength is approximated by a maximum entropy method using its first four statistical moments. The principle of maximum entropy [19,20] is an more powerful alternative for determining a probability distribution when certain information is given. For the current study, these certain information is available in the form of statistical moments if PEM is used to calculate the first four statistical moments of bearing strength of a composite joint. In fact, a set of finite statistical moments is insufficient to uniquely determine the underlying probability distribution in probability theory. Following the principle of maximum entropy, however, one can select a distribution amongst all possible distributions, which is consistent with the available partial information and contains minimum arbitrary assumption of information. Additionally, it is the only unbiased distribution [19,20]. The probabilistic methods evolved from the principle of maximum entropy have been successively applied in probabilistic engineering mechanics [21–25].

The remaining sections of this paper are organized as follows. After presenting the problem of probabilistic bearing strength of composite pinned joint in section 2, the principle of maximum entropy and the computational algorithm for Lagrange multipliers given statistical moment constraints are discussed in section 3. A demonstration example involved T300/QY8911 composite laminates is shown in Section 4 to examine the capacity of the proposed framework, before the paper is concluded in section 5.

2. Problem statement

It has been observed that the basic failure modes of composite joints are bearing, tension and shear-out failures. From these three failure modes only bearing damage produces a progressive failure, thus, bearing failure is much desirable to be observed when designing reliable composite joints in practical engineering. In this section, the deterministic bearing strength prediction using finite element (FE) method is presented and the associated uncertainty quantification issues are discussed.

2.1. Deterministic finite element analysis of pin joint in composite plate

2.1.1. Finite element model

Consider a rectangular composite laminated plate of length L, width W, and thickness t with a hole of diameter D. The hole is at a distance E from the free edge of the plate. The configuration of the composite plate is shown in Fig. 1. The structural analysis of the pin joint is accomplished via commercial FE code ABAQUS [10]. The 4-node continuum plane stress element (CPS4) in ABAQUS is selected to simulate the contact problem between the hole and the pin because the shell element in ABAQUS cannot simulate the in-plane contact problem, as suggested by Dano et al. [4,5]. Instead of modeling each ply individually, all plies with the same fiber orientation are modeled using a single layer of CPS4 elements so that only four distinct layers of CPS4 element are required for a frequently-used composite plate involved 0°, ±45° and 90° fiber directions. It should be noted that all layers possess the same nodes but different numeration is used to distinguish each other when assembling elements. The pin is modeled as a rigid surface since the failure of pin is not as important as the failure of composite plate in practice. On the other hand, the hole edge is considered as a deformable surface, and then a contact pairs can be defined between two surfaces in ABAQUS. In view of the geometrical symmetry of the composite plate, one half EF model of the plate is created as shown in Fig. 2. Since a parametric FE model is more preferable in a probabilistic analysis, which involved repeated FE analysis, an input file for the 2D FE model of the above-described composite pinned joint has been developed in the ABAQUS software. When implementing repeated structural analysis, an advanced computer language can be used to rewrite the value of a parameter in the input file, such as modulus of elasticity, and submit the new job to ABAQUS in the batch mode. By doing this, the work of probabilistic strength analysis can process as an extension of deterministic analysis in a non-intrusive manner.

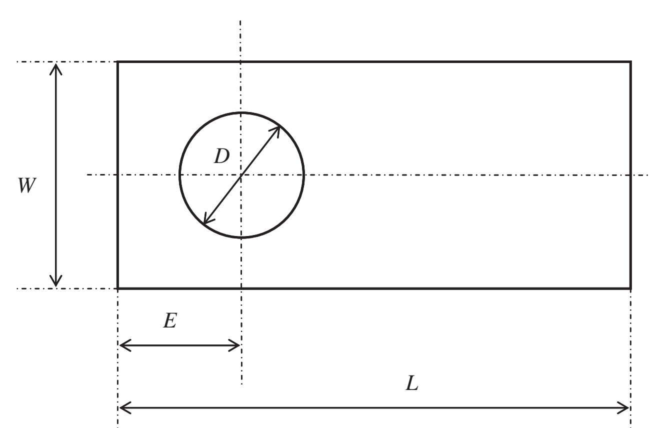

Fig. 1. The configuration of composite plate.