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Research Paper

A coupled thermo-hydro-mechanical finite element formulation for curved beams in two-dimensions

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ABSTRACT

To enable the use of beam elements in the modelling of coupled thermo-hydro-mechanical (THM) geotechnical problems, a fully coupled and robust THM formulation is required. This paper presents such a formulation which allows both fluid flow and heat transfer along a 2D curved beam, while ensuring compatibility with coupled THM solid elements commonly used to discretise soils. Verification exercises and application with the proposed coupled beam element are carried out to demonstrate its satisfactory behaviour. The results of these analyses are compared against closed form solutions, solutions obtained using solid elements, and field measurements, showing an excellent agreement.

1. Introduction

Coupled thermo-hydro-mechanical (THM) interactions in porous media (e.g. soils) have been the focus of recent research as many geotechnical engineering problems, such as geothermal energy systems or storage of high-level radioactive waste, involve temperature variations [1]. To adequately simulate the coupled THM behaviour of porous media in a finite element (FE) analysis, a number of coupled THM formulations have been developed in the literature [2–12]. However, most of the existing formulations have only been applied to solid elements, which are commonly used to discretise the porous medium. In theory, it is possible to use these coupled THM solid elements to model structural components such as thermo-active retaining walls, piles, tunnel linings or heat exchanger pipes. In practice, however, this may result in either a very large number of elements or elements with unacceptably large aspect ratios [13]. To overcome these shortcomings and adequately model the interaction between the porous medium and the structure in a coupled THM FE analysis, a special type of beam element, which can accommodate both consolidation and heat transfer, is required. Additionally, beam elements are frequently a preferred element type for discretising structural components, as they are formulated directly in terms of structural forces, rather than the component stresses, hence being easier to use.

Russell [14] adopted a three-noded line element, described as a drainage element, to model a vertical drain in an isothermal consolidation FE analysis. However, as this coupled hydro-mechanical (HM) line element has a zero cross sectional area, it becomes problematic when applying it to model the mechanical behaviour of structural components. Moreover, the off-diagonal coupling terms in the HM formulation of the line element are zero, which prevents using the line element to simulate the fully coupled HM behaviour. To the authors' knowledge, coupled THM formulations for beam elements have not been published elsewhere.

The mechanical formulation for a two-dimensional (2D) curved beam element developed by Day and Potts [15] is used here as a basis from which to extend the formulation to account for the effects of both pore fluid pressure and temperature changes, resulting in a coupled THM formulation. The authors have recently developed a new fully coupled THM formulation for 2D and 3D solid elements [12]. In order to ensure compatibility between solid elements and beam elements in an FE analysis, the new THM formulation for 2D beam elements adopts the same development approach and assumptions. Therefore, the proposed beam elements can be used to model the behaviour of either a two-phase porous medium intrinsically, or a single-phase material (e.g. concrete structures) by setting appropriate material properties to each phase. Moreover, as the formulation of each coupling system is developed individually, it is possible to disable any of the three systems (thermal, hydraulic and mechanical) if they are not active in the analysis, thus enabling the coupled THM beam element to be used in coupled HM, TM or TH, or uncoupled mechanical, hydraulic or thermal analyses. This flexible formulation for the coupled THM 2D curved beam element has been successfully implemented into the bespoke FE

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Nomenclature

A	cross section area
[B]	matrix containing derivatives of the displacement interpolation functions
C_{pf}	specific heat capacity of the pore fluid
C_{ps}	specific heat capacity of the solid skeleton
[D]	constitutive matrix
e	void ratio
E	Young's modulus
G	shear modulus
$h_{f.l}$	hydraulic head of the beam
I	moment of inertial
k	shear correction factor
K_f	bulk modulus of the pore fluid
$k_{w,l}$	hydraulic conductivity of the beam
$k_{T,l}$	conductivity of the beam
l	distance along the beam
[N]	matrix of displacement interpolation functions
$[N_p]$	matrix of pore fluid pressure interpolation functions
$[N_T]$	matrix of temperature interpolation functions
$[N'_p]$	matrix containing derivatives of the pore fluid pressure interpolation functions
$[N'_T]$	matrix containing derivatives of the temperature interpolation functions
n	porosity
p_f	pore fluid pressure
Q^f	pore fluid sources and/or sinks
Q^T	heat sources and/or sinks
q_d	heat diffusion
q_c	heat convection
R	radius of curvature
r_o	circumferential radius
T	temperature of the porous medium
T_r	reference temperature
t	time
u_l	displacement tangential to the beam
dV	volume of the beam
$v_{w,l}$	seepage velocity along the beam
w_l	displacement normal to the beam
α_1	time marching parameter
α_T	linear thermal expansion coefficient of the porous medium
$\alpha_{T,f}$	linear thermal expansion coefficient of the pore fluid
α_b	rotation from global to local coordinate directions
β_1, β_2	time marching parameter
γ	shear strain
γ_f	specific weight of the pore fluid
ε	total strain
ε_l	axial strain
ε_v	volumetric strain
ε_{vT}	thermal volumetric strain
ε_Ψ	circumferential membrane strain
ε_σ	mechanical strain
ε_T	thermal strain
ρ_f	densities of the pore fluid
ρ_s	densities of the solid skeleton
μ	Possion's ratio
θ	cross section rotation
χ_l	bending strain

software ICFEP [13], to demonstrate the element's performance in a number of verification exercises, as well as case studies. In the present formulation, displacement, pore fluid pressure and temperature are the nodal degrees of freedom.

2. Development and implementation of a coupled THM formulation for a 2D curved beam element

2.1. Strain definition and mechanical equilibrium of the beam element

2.1.1. Strain definition

The beam element developed by Day and Potts [15] was designed as a special type of zero-thickness line element for modelling structural components, e.g. retaining walls and tunnel linings, in a 2D analysis. It is formulated directly in terms of bending moments, axial and shear forces and their associated strains, under the consideration that the dimension of the structural component in the in-plane transversal direction is zero.

Fig. 1 shows a general curved 2D beam element in the global coordinate system x-y. The corresponding degrees of freedom are marked as displacements u and v in the x and y directions respectively, and the cross section rotation θ.

The strains of the 2D curved beam element have to be defined in a local coordinate system, with directions tangential and normal to the beam [15]:

Axial (longitudinal) strain

$$\varepsilon_l = \frac{du_l}{dl} + \frac{v_l}{R} \tag{1}$$

Bending strain

$$\chi_l = -\frac{d\theta}{dl} \tag{2}$$

Shear strain

$$\gamma = -\frac{u_l}{R} + \frac{dv_l}{dl} - \theta \tag{3}$$

where l is the distance along the beam, R is the radius of curvature, u_l and v_l are the displacements tangential and normal to the beam, respectively. The angle α_b in Fig. 1 is the rotation from global to local coordinate directions. For axisymmetric analysis, the definition of additional terms is required:

Circumferential membrane strain

$$\varepsilon_\Psi = \frac{u_l\cos\alpha_b - v_l\sin\alpha_b}{r_0} \tag{4}$$

Circumferential bending strain

$$\chi_\Psi = -\frac{\theta\cos\alpha_b}{r_0} \tag{5}$$

where r_0 is the circumferential radius.

2.1.2. Mechanical equilibrium

When only the mechanical behaviour of the beam is taken into account, the constitutive relation between the strain terms and the

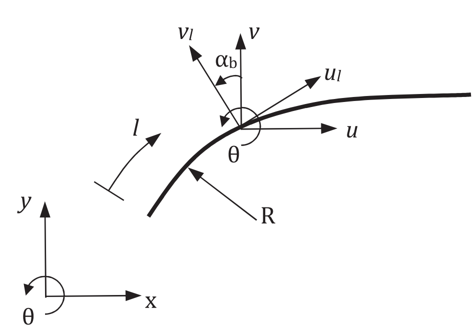

Fig. 1. Definition of terms and axes of the beam element.