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Research Paper

Vertical vibrations of rigid foundations of arbitrary shape in a multi-layered poroelastic medium

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ABSTRACT

This paper presents dynamic response of rigid foundations of arbitrary shape embedded in a multi-layered poroelastic medium subjected to time-harmonic vertical loading. The contact surface between foundation and poroelastic medium is smooth, and either permeable or impermeable. This dynamic interaction problem is investigated by employing a discretization technique and an exact stiffness matrix scheme. Selected numerical results are shown to portray the influence of various parameters such as foundation shape, embedded depth, and poroelastic effects on vertical compliances of various rigid foundations in multi-layered poroelastic media. In addition, vertical vibrations of mudmat foundations on water-saturated layered soils are also presented.

1. Introduction

The study of dynamic interaction between foundation and supporting soil medium is of considerable interest in geotechnical engineering and earthquake engineering since its results are very useful in the analysis and design of foundations subjected to dynamic loading (e.g., vibrating machines, high-speed railway, subway, blast load, etc.). In the past, the classical problems of dynamic interaction between rigid foundations and a homogeneous elastic medium were studied by many researchers who employed a variety of analytical and numerical techniques. Examples of past studies include strip foundations [1,2]; circular foundations [3–5]; rectangular foundations [6,7]; and foundations of arbitrary-shaped [8–10]. Those existing studies, however, considered the supporting soil medium as a single-phase elastic solid. Geomaterial is often a two-phase material consisting of an elastic solid with voids filled with water, known as a poroelastic material, which is considered to be more realistic representation of natural soils and rocks than the single-phase elastic solid. The theory of wave propagations in a poroelastic material was presented by Biot [11,12] by adding the inertia terms to his three-dimensional consolidation theory [13]. For the past thirty years, Biot's poroelastodynamics theory has widely employed by many researchers to study various soil-structure interaction problems involving homogeneous poroelastic media and various foundations including strip [14,15] and circular [16–19] foundations. In addition, vertical vibrations of a rigid rectangular foundation resting on a homogeneous poroelastic half-space were also presented by Halpern and Christiano [20].

It is well known that natural soil profiles are normally layered in character. Studies related to dynamic interaction between vertically loaded foundations and a multi-layered poroelastic medium are very limited when compared to the case of layered elastic media (e.g., [21–28]). Philippacopoulos [29] investigated vertical vibrations of a rigid circular foundation resting on a layered poroelastic half-space. Dynamic response of a rigid strip bonded to a multi-layered poroelastic medium was considered by Senjuntichai and Rajapakse [30], who employed the discretization technique and an exact stiffness matrix method [31]. In the exact stiffness matrix scheme, stiffness matrices of all layers and underlying half-space are derived explicitly from the general solutions presented in their previous paper [32]. The global equation is then obtained from the continuity of displacements and traction at all layer interfaces. Subsequently, dynamic interaction between multiple strips and a multi-layered poroelastic half-plane was also studied by Senjuntichai and Kaewjuea [33]. In addition, Senjuntichai and Sapsathiarn [34] presented vertical vibrations of an elastic circular plate embedded in a multi-layered poroelastic medium by employing the exact stiffness matrices derived for axisymmetric deformations. In practical situations, foundations are constructed in various shapes, such as circular, triangular, and most of all, rectangular. For certain applications, such as mudmat foundations for offshore structures, the footings are typically constructed with openings. A review of literature indicates that existing studies related to dynamic interaction between rigid foundations and multi-layered poroelastic media have been limited to plane strain and axisymmetric deformations.

This paper presents vertical vibrations of an arbitrary-shaped foundation of width $2H$ embedded at a depth h in a multi-layered

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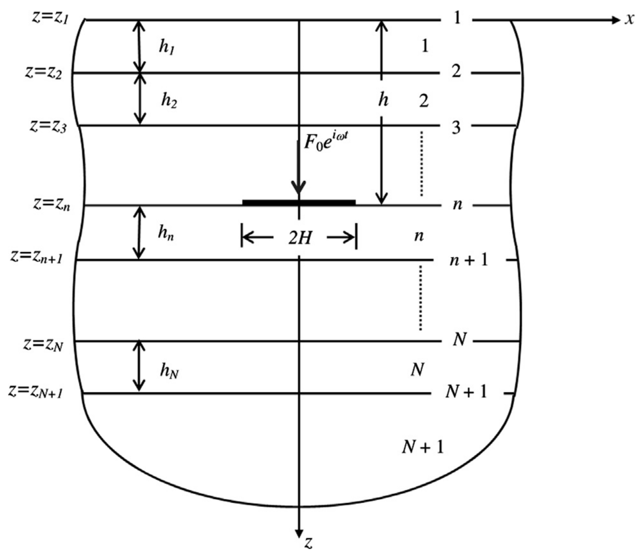

Fig. 1. Rigid foundation in a multi-layered poroelastic half-space.

poroelastic half-space as shown in Fig. 1. The foundation is assumed to be rigid and massless, and undergoing time-harmonic vertical displacement. Each layer of the multi-layered half-space is governed by Biot's theory of poroelastodynamics. The contact surface between the foundation and the layered medium is assumed to be smooth and can be either fully permeable or impermeable. The discretization technique [8] is adopted in order to solve this interaction problem. Based on this technique, the contact area under the foundation is divided into a number of square elements, over which the contact traction is assumed to be constant within each element. The unknown contact traction and fluid pressure (in the case of impermeable foundation) within each discretized element are then solved from the flexibility equations based on the influence functions, which are the fundamental solutions of a multi-layered poroelastic half-space subjected to vertical loading and fluid pressure. Those influence functions are determined by employing the exact stiffness matrix method. A computer program based on the proposed scheme has been developed, and comparisons with existing solutions of rigid foundations on elastic and poroelastic media are shown to verify its accuracy. Selected numerical results are presented to investigate the influence of various parameters on vertical compliances of rigid foundations in a multi-layered poroelastic medium. In addition, the application of the present solution scheme to investigate vertical vibrations of mudmat foundations is also demonstrated.

2. Basic equations

Consider a poroelastic half-space with a Cartesian coordinate system (x,y,z) defined such that the z-axis is perpendicular to the free surface as shown in Fig. 1. Let $u_i(x,y,z,t)$ and $w_i(x,y,z,t)$ denote the average displacement of the solid matrix and the fluid displacement relative to the solid matrix in the i-direction $(i = x,y,z)$, respectively. The constitutive relation of a homogeneous poroelastic material can be expressed according to Biot's theory of poroelasticity [13] by using the standard indicial notation as

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk} - \alpha\delta_{ij}p, \quad i,j = x,y,z \tag{1a}$$

$$p = -\alpha M\varepsilon_{kk} + M\zeta \tag{1b}$$

In the above equations, σ_{ij} is the total stress component of the bulk material; ε_{ij} is the strain component of the solid matrix, which is related to the displacement u_i as in ideal elasticity; μ and λ are Lame' constants of the bulk material; δ_{ij} is the Kronecker delta; p is the excess pore fluid pressure (suction is considered negative); and ζ is the variation of fluid content per unit reference volume, defined as $\zeta = -w_{i,i}$. In addition, α and M are Biot's parameters accounting for compressibility of the two-phased material.

The equations of motions in the absence of body forces (solid and fluid) and a fluid source can be written in terms of u_i and w_i as [35]

$$\mu u_{i,jj} + (\lambda + \alpha^2 M + \mu)u_{j,ji} + \alpha M w_{j,ji} = \rho\ddot{u}_i + \rho_f\ddot{w}_i \tag{2a}$$

$$\alpha M u_{j,ji} + M w_{j,ji} = \rho_f\ddot{u}_i + m\ddot{w}_i + b\dot{w}_i \tag{2b}$$

where ρ and ρ_f are the mass densities of the bulk material and the pore fluid, respectively; m is a density-like parameter that depends on ρ_f and the geometry of the pores; and b is a parameter accounting for the internal friction due to the relative motion between the solid matrix and the pore fluid. In addition, the superscript dot denotes the derivative with respect to time. In the present study, the motion under consideration is assumed to be time-harmonic with the factor of $e^{i\omega t}$, where ω is the frequency of motion and $i = \sqrt{-1}$. The term $e^{i\omega t}$ is henceforth suppressed from all expressions for brevity.

Eq. (2) can be solved by using Helmholtz representation for a displacement vector field and applying the double Fourier integral transform with respect to the horizontal coordinates. The double Fourier integral transforms with respect to the two horizontal coordinates x and y can be expressed as [36]

$$\overline{f}(k_x,k_y,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y,z)e^{-\mathrm{i}k_x x-\mathrm{i}k_y y}dxdy \tag{3a}$$

in which k_x and k_y denote the wave numbers associated with the x and y coordinates, respectively. The inverse relationship is given by

$$f(x,y,z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{f}(k_x,k_y,z)e^{\mathrm{i}k_x x+\mathrm{i}k_y y}dk_x dk_y \tag{3b}$$

It can be shown that the general solutions can be expressed in the frequency-wave number domain in the following matrix form:

$$\mathbf{v}(k_x,k_y,z) = \mathbf{R}(k_x,k_y,z)\mathbf{c}(k_x,k_y) \tag{4a}$$

$$\mathbf{f}(k_x,k_y,z) = \mathbf{S}(k_x,k_y,z)\mathbf{c}(k_x,k_y) \tag{4b}$$

where

$$\mathbf{v}(k_x,k_y,z) = [\mathrm{i}\overline{u}_x\ \ \mathrm{i}\overline{u}_y\ \ \overline{u}_z\ \ \overline{p}]^T \tag{5a}$$

$$\mathbf{f}(k_x,k_y,z) = [\mathrm{i}\overline{\sigma}_{xz}\ \ \mathrm{i}\overline{\sigma}_{yz}\ \ \overline{\sigma}_{zz}\ \ \overline{w}_z]^T \tag{5b}$$

$$\mathbf{c}(k_x,k_y) = [A\ \ B\ \ C\ \ D\ \ E\ \ F\ \ G\ \ H]^T \tag{5c}$$

and $A_i(k_x,k_y)$ to $H_i(k_x,k_y)$ are the arbitrary functions that can be determined by employing appropriate boundary and continuity conditions. The matrices $\mathbf{R}$ and $\mathbf{S}$ are given explicitly in the Appendix A.

3. Influence functions and exact stiffness matrix method

The analysis of dynamic interaction problem shown in Fig. 1 by using the proposed discretization technique requires a set of displacement influence functions of a multi-layered poroelastic half-space under vertical loading and fluid pressure applied over a square area. An exact stiffness matrix method is employed to determine the required influence functions. A brief outline of the stiffness matrix scheme is presented here, and more details on the exact stiffness matrix scheme are given elsewhere [31,34,37].

Consider a multilayered half-space consisting of N poroelastic layers overlying a poroelastic half-space with layers and interfaces being numbered as shown in Fig. 1. For an nth layer $(n = 1, 2, 3, \ldots, N)$, the