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Research Paper

A coupled hydro-mechanical constitutive model for unsaturated frictional and cohesive soil

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ABSTRACT

The present study has developed a constitutive model for both saturated and unsaturated soils. The model considers the effect of mechanical and hydraulic loading on the soil behavior in critical state framework. The mechanical part has separate mechanisms, deviatoric mechanism taking advantage of multi-yield surface plasticity, and isotropic mechanism with classical plasticity formulation. The water retention part is based on the bounding surface concept. Comparison of experimental data and model predictions for saturated and unsaturated states in frictional and cohesive soils, validates the model performance considering soil behavior under different drainage and loading conditions by using one set parameters.

1. Introduction

Both saturated and unsaturated states of soil media must be considered in analysis and design of geotechnical engineering projects, because a large portion of the world is covered by unsaturated soil which can become saturated with changes in the climate or groundwater conditions [1]. Gas and liquid phases exist in unsaturated soil. The pressure difference in these phases creates suction, which increases the strength and stiffness of the soil compared to the saturated state.

The suction is related to the degree of saturation by means of a relationship that is expressed through a water retention curve (WRC). Experimental results show that there is no unique relationship between suction and the degree of saturation and it depends on wetting-drying path. This phenomenon is called hydraulic hysteresis [2].

Because of widely spreading unsaturated soil in the world, it is essential to consider their stress-strain behavior in analysis and design of earth structures. An appropriate constitutive model is required to do this. Several models are suggested for unsaturated soils up to now. Some models use independent stress state variables as proposed by Fredlund and Morgenstern [3]. These constitutive models use net stress (total stress minus gas pressure) and suction to model unsaturated soil response. The Barcelona Basic Model (BBM) [4] is an example of such models, but these models have some disadvantages [5]. On the other hand, some researchers, such as Loret proposed an effective stress based model [6]. These models consider the influence of suction as a hardening factor in the constitutive relationship. Some effective stress-based models are in bounding surface or generalized plasticity framework [7–11].

A characteristic behavior of unsaturated soils is coupling between the soil loading history and WRC that has been proven by experimental data [2,12]. Mechanical loading changes the water content and suction and also change in water content or suction causes variations in the stress and deformation of soil specimen. Consideration of such behavior when modeling unsaturated soil behavior is necessary, but little attention has been devoted to this issue. Many models do not consider hydraulic behavior or hydraulic hysteresis [13]. Some models neglect variations in suction and degree of saturation during loading [11]. Models for coupled behavior are also limited and some only are proper to the isotropic stress state [14]. Furthermore, wide ranges of constitutive models are only suitable for cohesive or frictional soils. No suitable model is available that models both frictional and cohesive unsaturated soil behavior. Most of the models are useful for cohesive soil and are in the Cam Clay constitutive model family. These models do not consider the Bauschinger effect and are not suitable for predicting the cyclic behavior of soil [15]. In geomaterials, such as soils, occurring initial yielding and plastic deformation in one direction, reduces yielding stress in the opposite direction, which is called Bauschinger effect. This effect describes the anisotropy of the yield stress after plastic deformation that is observed in soils under cyclic loading.

Based on issues mentioned above, the present study has developed an effective stress based coupled hydro-mechanical constitutive model for both frictional and cohesive soils in saturated and unsaturated states. To predict the behavior under different states and loading conditions using only a single calibration, model formulation has been proposed using a state parameter within the critical state framework. For the mechanical part, two separate mechanisms are used for

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deviatoric and isotropic loadings. Multi-yield surface plasticity, which has shown good performance for predicting the behavior of the soil, was employed as a deviatoric mechanism and conventional plasticity was used for the isotropic part. For the water retention (hydraulic) part, the bounding surface concept is used to model the wetting and drying paths. The proposed model is validated using a large number of experimental results for drained and undrained conditions under cyclic and monotonic loadings for both saturated and unsaturated soils.

2. Notation

The compressive stresses and strains are assumed to be positive. "p" denotes the mean effective stress and "S" denotes the deviatoric stress tensor (Eq. (1)). Suction is used as another stress variable to depict aspects of unsaturated soil such as elastoplastic response under the wetting path. For triaxial stress, "$p = \frac{\sigma_1 + 2\sigma_3}{3}$" denotes the mean effective stress and "$q = \sigma_1 - \sigma_3$" denotes the deviator stress. In general stress space, these stresses are defined as:

$$p = \frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3}{3}; \quad \mathbf{S} = \boldsymbol{\sigma} - p\boldsymbol{\delta} \tag{1}$$

δ in this equation is Kroneker delta.

3. Effective stress in Unsaturated Soil

The model is in effective stress space as presented by Bishop [16]. This definition is commonly accepted (e.g. Loret et al. [17]). For Bishop effective stress definition, the effective stress is defined by participation of the solid, liquid (water) and gas (air) phases as shown in Eq. (2):

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w) = (\sigma - u_a) + \chi \cdot S; \quad \chi = \textit{effective stress parameter} \tag{2}$$

where u_a and u_w are the air pressure and water pressure in the soil, respectively, and S is the matric suction. Suction participation in effective stress is controlled by effective stress parameter χ, which equals 1 in the saturated state and zero in the dry state.

The selection of χ is decisive in this equation. Researchers, such as Borja [18], have provided different effective stress parameters in their theoretical studies. Others have determined this parameter using experimental results. Khalili and Khabaz [19] proposed a relationship to determine this parameter based on laboratory results that was subsequently corrected by Russell and khalili [20] and is shown in Eq. (3) as:

$$\chi = \begin{cases} 1 & S \leqslant S_e \\ \left(\frac{S}{S_e}\right)^{\gamma} & S_e < S \leqslant 25S_e \\ 25^{0.45}\left(\frac{S}{S_e}\right)^{-1} & S > 25S_e \end{cases} \tag{3}$$

where S_e is air entry (or exit) and γ is the model parameter that is assumed to equal −0.55. Because of its simplicity and workability, this equation has been used in this study.

The use of effective stress in constitutive models reduces the number of parameters and simplifies the hydro-mechanical coupling behavior. In addition, the critical state stress ratio is independent of suction, as confirmed by various researchers [21,22]. Fig. 1 shows the critical state stress ratio based on the experimental results from Maâtouk et al. [23] in effective stress space and confirms a unique line for both saturated and unsaturated cases.

4. Mechanical part of the model

4.1. Elastic behavior

Isotropic elastic behavior is assumed for the elastic response of the model. Based on this assumption, elastic volumetric and deviatoric strains are expressed as shown in Eq. (4):

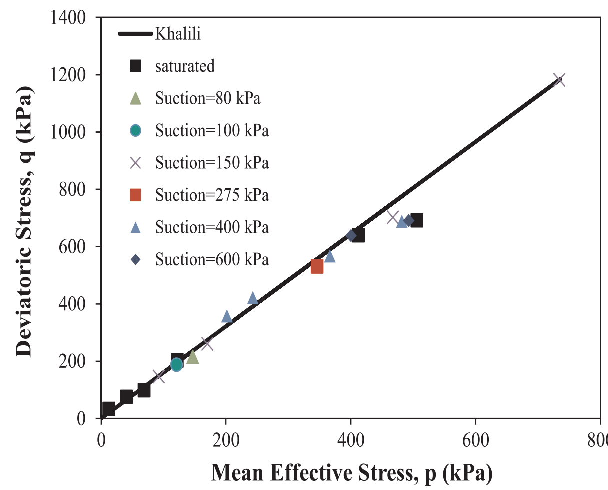

Fig. 1. Critical state line on *p*–*q* plane for different suction [23].

$$\dot{\varepsilon}_v = \frac{\dot{p}}{B}; \quad \dot{\varepsilon}_d = \frac{\dot{\mathbf{S}}}{2G} \tag{4}$$

where *B* and *G* are bulk and shear modulus, respectively. The elastic moduli are a function of the mean effective stress and void ratio and are defined in Eq. (5) [24]:

$$G = G_1\left(\frac{p}{p_{ref}}\right)^{0.5}\left(\frac{2.17 - e}{1 + e}\right)$$
$$B = B_1\left(\frac{p}{p_{ref}}\right)^{0.5}\left(\frac{2.17 - e}{1 + e}\right) \tag{5}$$

where p_{ref} is the reference mean effective stress in which B_1 and G_1 are evaluated and *e* is the current void ratio. Note that the effect of mean effective stress on the elastic moduli is more than the effect of void ratio. Suction does not affect elastic moduli directly and its influence on these moduli is simulated by using unsaturated effective stress.

4.2. Deviatoric mechanism

4.2.1. Yield surface

The deviatoric mechanism of model is based on multi-yield surface plasticity framework. The yield surfaces are conical and generate multiple nested circles in the deviatoric plane. To consider the effect of cohesion in cohesive soil, the apexes of the yield surfaces are shifted along the hydrostatic axis using p_{1-0}. This parameter is only considered for cohesive materials and is formulated in Section 4.5. The general form of the deviatoric yield surfaces is as follows:

$$F_1 = \frac{3}{2}(\mathbf{S} - \boldsymbol{\alpha}p):(\mathbf{S} - \boldsymbol{\alpha}p) - m^2p^2 \tag{6}$$

$$p = p - p_{1-0} \tag{7}$$

where $\boldsymbol{\alpha}p$ is the tensor denoting the location of the center of each yield surface and *m* is dependent on the volumetric strain that controls the size of the yield surfaces. In triaxial stress space, Eq. (6) becomes equal to Eq. (8) and represents two lines in the *p-q* plane.

$$F_1 = (q - \alpha p)^2 - m^2p^2 \tag{8}$$

where $\alpha = \alpha_1 - \alpha_3$.

α denotes the position of the center of yield surface in triaxial stress space. Graphical representations of yield surfaces in general and the triaxial stress spaces are shown in Fig. 2.

4.2.2. Flow rule

The flow rule for the deviatoric yield mechanism is written in the form of Eq. (9):