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Research Paper

A modified multi-yield-surface plasticity model: Sequential closest point projection method

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ABSTRACT

This paper presents a modified multi-yield-surface (MMYS) plasticity model employing a 'sequential closest point projection' method that is consistent with the discretization of the backbone curve of a multi-yield-surface (MYS) model. Compared with existing MYS models, the newly developed MMYS model eliminates the main sources of numerical errors caused by inconsistencies between the model and discretized backbone curve, which significantly improves the numerical stability and convergence rate of the Newton–Raphson (N–R) iterative process at a structural level. Furthermore, tangent operators consistent with the integration algorithm are derived, preserving the quadratic rate of convergence in the N–R process. The MMYS model has been implemented in OpenSees, an open system for earthquake engineering simulation, and verified by two application examples. The N–R process is more stable and generally converges faster when using the MMYS model rather than a MYS model. The advantages of the MMYS model become more remarkable when the tolerance used in convergence criterion is tightened, the external pushover force or seismic excitation is increased, or the time/load step size is enlarged, regardless of the number of yield surfaces used in the models. This study enhances the capacity of the existing MYS model that is widely used in geotechnical or soil–structure interaction (SSI) problems.

1. Introduction

In geotechnical engineering, the multi-yield-surface (MYS) plasticity model has been widely used to simulate the nonlinear cyclic behaviour of soils under static pushover or dynamic loading conditions. This model has been developed, improved and applied to various geotechnical and soil–structure interaction (SSI) problems by Iwan [1], Mroz [2–4], Prevost [5–7], Elgamal [8,9], and other co-workers [10,11]. In the MYS plasticity model, the smooth backbone curve is discretized using a set of piecewise linear approximations, with each line segment corresponding to a single yield surface of constant size (i.e., no isotropic hardening) in the stress space. A field of plastic moduli [2,5] is employed to control the flowing and hardening behaviour for each yield surface to achieve a larger representation of the material's plastic behaviour under cyclic loading conditions. The flow rule corresponding to each yield surface is the same as that used in a linearized J_2 plasticity model [6], while the hardening law is based on the rule that the surfaces cannot cross each other when they touch and push each other. During each Newton–Raphson (N–R) iteration process of a load step, it is not possible to know in advance which and how many yield surfaces will be touched (or activated) given a strain increment. Thus, the kernel algorithm used in the existing MYS plasticity model is based on a 'single predictor followed by multiple correctors' numerical scheme that only uses one elastic predictor followed by several plastic correctors. The correction process is repeated until the corrected stress lies inside the next larger yield surfaces. However, this algorithm is inconsistent with the discretization method of the backbone curve and leads to the accumulation of numerical errors. More importantly, this algorithm might lead to numerical instability and cause non-convergence of the N–R iterative process at the structural level when the system is highly nonlinear. To overcome these problems, a modified multi-yield-surface (MMYS) model is presented in this paper and applied to a pressure independent MYS J_2 plasticity model. The MMYS model uses a sequential closest point projection method, or a sequential 'elastic-predictors and plastic-corrector' method. The total strain increment in the current step is subdivided into sequential sub-increments, such that the increased stress corresponding to each sub-increment lies exactly on the next larger yield surface and can be obtained using a single 'elastic-predictor and plastic-corrector' process. Many methods can be used to divide the total strain increment into sub-increments,

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and in this paper, the direction of each sub-increment is kept the same as the direction of the total strain increment for convenience. Furthermore, a safeguarded Newton algorithm is employed to obtain the length of each sub-increment. The 'sequential closest point projection' method presented in this paper can be applied to pressure independent and pressure dependent MYS models. However, this method is not applicable to other types of models that do not sequentially correct for stress, like the Cap plasticity model. The MMYS model is consistent with a linearization of the backbone curve in the sense that each single closest point projection does not lead to the numerical errors as in existing MYS models, which improves the numerical stability and increases the convergence rate of the N–R process when using the MMYS model. The MMYS model used in this paper (i.e., pressure independent MYS J_2 plasticity model) can be applied to other MYS models defined in the literature [12–19]. The newly developed MMYS model has been implemented in OpenSees to simulate geotechnical or SSI problems. OpenSees [20] is an open source software framework for advanced modelling and the simulation of structural and/or geotechnical problems developed under the auspice of the Pacific Earthquake Engineering Research (PEER) Center (http://peer.berkeley.edu) and has been widely used.

Furthermore, the consistent tangent moduli (also called the algorithmic tangent moduli) of the MMYS model are derived, and software is implemented in OpenSees to achieve efficient computation. Consistent tangent moduli are important for preserving the asymptotic quadratic convergence rate of the N–R process at the structural level. Detailed illustrations of consistent tangent moduli can be found in the literature presented by Simo, Taylor and other researchers [21,22]. These tangent moduli are consistent with the numerical integration scheme of the material constitutive model, which is obtained by differentiating the incremental constitutive equations ($\Delta\boldsymbol{\sigma} = \Delta\boldsymbol{\sigma}(\Delta\boldsymbol{\varepsilon})$) with respect to the total incremental strains $\Delta\boldsymbol{\varepsilon}$ instead of differentiating the rate constitutive equations ($\dot{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}}(\dot{\boldsymbol{\varepsilon}})$) (with respect to the $\dot{\boldsymbol{\varepsilon}}$ strain rate (i.e., the classical continuum tangent moduli). A three-dimensional (3D) solid block subjected to quasi-static cyclic loading conditions, and a 3D pile–soil interaction system subjected to quasi-static and dynamic cyclic loading conditions are used as examples. The Newton–Raphson (N–R) iteration process at the structural level when using the newly developed MMYS model exhibits greater numerical stability than the existing MYS model. The advantages of the MMYS model over the MYS model become more remarkable when the tolerance used in the convergence criterion is tightened or the load step size increases, regardless of the number of yield surfaces used in the models. The results of this work enhance the capacity of the MYS model by improving the finite element analysis capacity when using the model to solve a wide class of geotechnical and SSI problems.

2. Mathematical expression of the multi-yield-surface J_2 plasticity material model

In this section, a mathematical expression of the constitutive pressure independent MYS J_2 plasticity material model (i.e., the yield surfaces, the flow rule and the hardening law) is summarized. The complete formulations for this model are described in the literature [10–12,23].

2.1. Multi-yield surfaces

In the context of the pressure independent MYS J_2 plasticity model, each yield surface is defined in the deviatoric stress space as follows:

$$f_m = \left\{\frac{3}{2}\left(\boldsymbol{\tau}-\boldsymbol{\alpha}^{(m)}\right):\left(\boldsymbol{\tau}-\boldsymbol{\alpha}^{(m)}\right)\right\}^{\frac{1}{2}} - K^{(m)} = 0 \quad (m = 1,2,3\ldots\mathrm{NYS}) \tag{1}$$

where $\boldsymbol{\tau}$ denotes the deviatoric stress tensor, m denotes the number of yield surfaces beginning from 1 to NYS, which is the total number of yield surfaces. The parameters $\boldsymbol{\alpha}^{(m)}$ and $K^{(m)}$ represent the back-stress tensor (the centre) and the size ($\sqrt{3/2}$ times the radius) of the mth yield surface $\{f_m = 0\}$, respectively. In geotechnical engineering, the nonlinear shear behaviour of the soil is described by the shear stress–strain backbone curve [24], as shown in Fig. 1(a). The experimentally determined backbone curve can be approximated using the following hyperbolic formula [11,25]:

$$\tau = \frac{G\gamma}{1+\gamma/\gamma_r} \tag{2}$$

where τ and γ denote the octahedral shear stress and shear strain, respectively, and G is the low-strain shear modulus. Parameter γ_r is the reference shear strain defined as $\gamma_r = \frac{\gamma_{\max}\cdot\tau_{\max}}{G\cdot\gamma_{\max}-\tau_{\max}}$, where $\tau_{\max}$, the shear strength, is the shear stress that corresponds to the shear strain $\gamma = \gamma_{\max}$ (selected as sufficiently large so that $\tau_{\max} \approx \tau(\gamma = \infty)$) (Fig. 1). The stress–strain points used to define the piecewise linear approximation of the originally smooth backbone curve are defined such that their projections on the τ axis are uniformly spaced [10]. Within the MYS plasticity framework, the hyperbolic backbone curve used in Eq. (2) is replaced by a piecewise linear approximation, as shown in Fig. 1(b). Each line segment represents the domain of a yield surface $\{f_m = 0\}$ of size $K^{(m)}$ that is characterized by an elasto-plastic shear modulus $H^{(i)}$ for $i = 1,2\ldots\mathrm{NYS}$ [8–10]. A constant plastic shear modulus, $H'^{(i)}$, is defined for each yield surface $\{f_i = 0\}$.

2.2. Flow rule

An associative flow rule is used to compute the plastic strain increments. In the deviatoric stress space, the plastic strain increment vector lies along the direction normal to the yield surface at the stress point. In tensor notation, the plastic strain increment corresponding to each surface $\{f_i = 0\}$ is expressed as follows:

$$d\boldsymbol{\varepsilon}_i^p = \frac{\langle L_i\rangle}{H'^{(i)}}\mathbf{Q}_i \tag{3}$$

where the second-order unit tensor, $\mathbf{Q}_i$ is defined as $\mathbf{Q}_i = \frac{1}{Q_i}\frac{\partial f_i}{\partial\boldsymbol{\sigma}}$, in which $Q_i = \left\{\frac{\partial f_i}{\partial\boldsymbol{\sigma}}:\frac{\partial f_i}{\partial\boldsymbol{\sigma}}\right\}^{\frac{1}{2}}$, represents the plastic flow direction normal to the yield surface $\{f_i = 0\}$ at the current stress point. The plastic loading function parameter, L_i in Eq. (3), is defined as the projection of the stress increment vector $d\boldsymbol{\tau}$ in the direction normal to the yield surface, i.e., $L_i = \mathbf{Q}_i : d\boldsymbol{\tau}$. The symbol $\langle\rangle$ in Eq. (3) denotes the MacCauley's brackets, which are defined such that $\langle L\rangle = \max(L, 0)$.

2.3. Hardening law

All yield surfaces ($\{f_i = 0\}$, $i = 1,2\ldots\mathrm{NYS}$) may be translated in the deviatoric stress space to the current stress point without changing the size of the yield surfaces (i.e., no isotropic hardening). In the context of MYS plasticity, the hardening law for the current (active) yield surface is different from that of the inner surfaces. For current active yield surfaces $\{f_m = 0\}$, the hardening law is generally governed by the philosophy that no overlap is allowed between the current and next yield surfaces. For the inner surfaces (hardening of the inner surface is performed after the current active yield surface $\{f_m = 0\}$ is updated), the consideration is that all inner yield surfaces $\{f_1 = 0\},\{f_2 = 0\},\ldots,\{f_{m-1} = 0\}$ must be updated such that all yield surfaces $\{f_1 = 0\}$ to $\{f_m = 0\}$ are tangent to each other at the current stress point, $\boldsymbol{\tau}$. A detailed description of the hardening law can be found in the literature [8–11].