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Rolling and sliding in 3-D discrete element models

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ABSTRACT

Rolling and sliding play fundamental roles in the deformation of granular materials. In simulations of granular flow using the discrete element method (DEM), the effect of rolling resistance at contacts should be taken into account. However, even for the simplest case involving spherical particles, there is no agreement on what is the best way to define rolling and sliding; various versions of definitions and calculations of rolling and sliding were proposed. Some even suggest that a unique definition for rolling and sliding is not possible. We re-check previous studies on rolling and sliding in DEMs and find that some researchers made a conceptual mistake when dealing with pure sliding between particles of different sizes. After considering the particle radius in the derivation of rolling velocity, the results yield a unique solution. Starting with clear and unique definitions of pure rolling and sliding, we present the detailed derivation and validate our results by checking two special cases of rolling. The decomposition of the relative motion is *objective*; that is, independent of the reference frame in which the relative motion is measured.

Introduction

Rolling and sliding are the two major local deformation mechanisms between contacting particles in granular materials. These mechanisms control the overall behavior of granular materials. Whereas sliding tends to dissipate energy through friction, rolling is a deformation mode available to the granular material to mitigate energy dissipation. In classical microscopic theories of strength and dilatancy of granular media, sliding is considered to be the dominant factor controlling the microscopic deformation of granular materials. For example, the shear strength and dilatancy of soils have been explained by pure sliding (Horne, 1965, 1969; Newland & Allely, 1957; Rowe, 1962; Shodja & Nezami, 2003) while neglecting effects from particle rolling. However, it has been recognized that particle rotation and rolling between particles play important roles in the mechanical behavior of granular materials, especially in those composed of circular or spherical particles. From experiments (Skinner, 1969), rolling is observed to become dominant as inter-particle friction increases. Oda, Konishi, and Nemat-Nasser (1982) also reported from their biaxial compression tests that inter-particle rolling dominates the micro-scale deformation of granular media.

If the discrete element method (DEM) is used to analyze the behavior of granular materials (Cundall & Strack, 1979), the overall effect of grain rotation on the strength of shear bands and the amount of energy dissipation in granular material can be studied. Bardet et al. (Bardet, 1994; Bardet & Proubet, 1991) examined the structure of shear bands in granular materials by numerically simulating idealized granular media. They showed that particle rotations concentrate inside shear bands and found that rotations have significant effects on shear strength of granular materials. Alonso-Marroquin, Vardoulakis, Hermann, Weatherley, and Mora (2006) and Mora and Place (1998) showed that the rolling mode between particles leads to a significant reduction of macroscopic frictional dissipation, supporting the idea that rolling provides a possible mechanism for the heat-flow paradox in the study of earthquake dynamics (Henyey & Wasserburg, 1971; Lachenbruch & Sass, 1992).

If spherical particles are adopted in the DEM, the calculated macroscopic friction is limited to very low values because of excessive rolling of the particles, and local friction has a limited effect on the macroscopic shear strength (Oda et al., 1982). This situation can be improved using non-spherical particles (Ng, 2009; Salot, Gotteland, & Villard, 2009), or by introducing more complex contact laws including an additional rolling resistance (Iwashita & Oda,

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Nomenclature

$\mathbf{n}$	a unit vector pointing from the center of particle 1 to the center of particle 2
$\mathbf{r}_1, \mathbf{r}_2$	position vectors of particles 1 and 2
$\mathbf{r}_c$	position vector of the contact point between two particles
$\mathbf{r}_{p1}, \mathbf{r}_{p2}$	vectors from the particle center to the contact point
R_1, R_2	radii of particles 1 and 2
$\mathbf{v}_1, \mathbf{v}_2$	linear velocity vectors of particles 1 and 2
$\mathbf{v}_{1n}, \mathbf{v}_{2n}$	normal components of $\mathbf{v}_1$ and $\mathbf{v}_2$ along vector $\mathbf{n}$
$\mathbf{v}_{1t}, \mathbf{v}_{2t}$	tangential components of $\mathbf{v}_1$ and $\mathbf{v}_2$ perpendicular to vector $\mathbf{n}$
$\mathbf{v}_c$	contact velocity vector of the particle pair
$\mathbf{v}_{ct}$	tangential component of the contact velocity $\mathbf{v}_c$
$\mathbf{v}_1^c, \mathbf{v}_2^c$	material velocities of the contact point in two particles
$\mathbf{v}_{1t}^c, \mathbf{v}_{2t}^c$	tangential components of $\mathbf{v}_1^c$ and $\mathbf{v}_2^c$
$\mathbf{s}_1, \mathbf{s}_2$	objective velocities of particles 1 and 2
$\mathbf{s}_{1r}, \mathbf{s}_{2r}$	components of $\mathbf{s}_1$ and $\mathbf{s}_2$ that contribute to rolling velocity
$\mathbf{s}_{1s}, \mathbf{s}_{2s}$	components of $\mathbf{s}_1$ and $\mathbf{s}_2$ that contribute to sliding velocity
$\boldsymbol{\omega}_1, \boldsymbol{\omega}_2$	angular velocities of particles 1 and 2
$\boldsymbol{\omega}_{1n}, \boldsymbol{\omega}_{2n}$	normal components of $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ along vector $\mathbf{n}$
$\boldsymbol{\omega}_{1t}, \boldsymbol{\omega}_{2t}$	tangential components of $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ perpendicular to the vector $\mathbf{n}$
$\boldsymbol{\omega}_{rb}$	overall angular velocity of the common rotating reference frame with respect to the global system
$\boldsymbol{\omega}_{rbs}$	angular velocity of rigid-body spinning
$\boldsymbol{\omega}_{rbr}$	angular velocity of rigid-body rotation
$\boldsymbol{\omega}_{pt}$	angular velocity of pure twisting between two particles

Acronyms

RB	rigid-body motion: two particles move together as a single rigid body
RBT	rigid-body translation: the particle pair has a rigid-body translational motion
RBR	rigid-body rotation: two particles rotate together as a single rigid body
RBS	rigid-body spinning: two particles spin together as a single rigid body around the vector $\mathbf{n}$
PR	pure rolling: two particles rotate over each other in a gear-like fashion
PS	pure sliding: occurs when the only motion is both particles rotating with the same angular velocity
PT	pure twisting between two particles

1998; Plassiard, Belheine, & Donz, 2009). Rolling resistance models are widely used by DEM researchers. Iwashita and Oda (1998, 2000) noted that the conventional DEM could not reproduce the large voids and high rotational gradients observed in shear band experiments. They found that rolling resistance causes arching at the contacts, permitting the easy formation of voids in physical tests. Therefore they proposed a modified model of the conventional DEM that took rolling resistance into account. Tordesillas et al. (Tordesillas, Peters, & Muthuswamy, 2005; Tordesillas & Walsh, 2002) incorporated rolling resistance in the DEM and examined the influence of particle rotation and rolling resistance in the rigid flat-punch problem, and found that extensive particle rotations occur near the edges of punch where there are high stress concentrations. These rotations lead to dilatation in the region adjacent to the sides of the punch. Wang and Mora (2008) showed that when only normal forces are transmitted, or rolling resistance is absent, the laboratory tests of wing-crack extension cannot be reproduced.

Quantitative investigation of the effects of rolling and sliding using the DEM demands a clear and unambiguous definition and calculation of rolling and sliding deformation. In principle, the relative motion between two particles in contact can be decomposed into several independent components: relative motion in the normal direction, relative motion in the tangential direction, or sliding, relative rolling, and in the 3-D case, relative twisting. However, even for the simplest 2-D case involving circular particles, there is surprisingly no agreement on what is the best way to define rolling and sliding. Various versions of definitions and calculations of rolling and sliding were proposed (Ai, Chen, Rotter, & Ooi, 2011; Alonso-Marroquin et al., 2006; Bagi & Kuhn, 2004; Bardet, 1994; Bardet & Proubet, 1991; Iwashita & Oda, 1998; Jiang, Yu, & Harris, 2005; Kuhn & Bagi, 2004a, 2004b; Luding, 2008; Mohamed & Gutierrez, 2010; Tordesillas et al., 2005; Tordesillas & Walsh, 2002). Some sources directly contradict others, confusing researchers in the DEM field. This even leads some researchers to suggest that there is no unique way to define rolling displacement (Bagi & Kuhn, 2004).

Based on clear definitions of pure rolling and sliding, Wang, Alonso-Marroquin, Xue, and Xie (2015) derived the rolling and sliding components in a simple way. They found that a conceptual mistake had been made in some previous models when dealing with pure sliding. After correcting the mistake, Iwashita–Oda's derivation and subsequently others produced correct results. Hence, they argued that there is indeed a unique way to determine the rolling velocity in the 2-D case. Rolling and sliding in the general 3-D case are more complicated. Currently there are only a few 3-D rolling models discussed in the literature (Bagi & Kuhn, 2004; Kuhn & Bagi, 2004a, 2004b; Luding, 2008), which do not agree with each other. The motivation of this paper is to derive rolling and sliding velocities in the 3-D case and to resolve the inconsistencies in rolling velocities found in the literature. As the method used to derive the 2-D rolling velocity is not general and cannot be applied to the 3-D case directly, in this paper we study the 3-D rolling problem by adopting a general vectorial notation developed in the previous papers (Alonso-Marroquin et al., 2006; Bagi & Kuhn, 2004; Kuhn & Bagi, 2004a, 2004b; Luding, 2008). This method is strict, unique, and objective.

Problem statement

Fig. 1 shows the kinematics of two particles indexed by 1 and 2. During a time step from t to $t+\Delta t$, two particles 1 and 2, with radii R_1 and R_2, respectively, remain in contact. Let $\mathbf{r}_1$ and $\mathbf{r}_2$ be the position vectors of the two particles, $\mathbf{v}_1$ and $\mathbf{v}_2$ be their linear velocity vectors, $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ be their angular velocities. Now the question is: for any arbitrary 3-D case, is there a unique way to determine rolling and sliding components? If the answer is yes, how? To answer these questions, we first need to define rigid-body (RB) velocity, objective velocities, pure rolling, and sliding.

Definition

Rigid-body velocity and objective velocities

A general vectorial notation developed in previous papers (Alonso-Marroquin et al., 2006; Bagi & Kuhn, 2004; Kuhn & Bagi, 2004a, 2004b; Luding, 2008) is followed here. Let $\mathbf{n}$ be a unit vector pointing from the center of particle 1 to the center of particle 2:

$$\mathbf{n} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|} \approx \frac{\mathbf{r}_2 - \mathbf{r}_1}{R_1 + R_2}, \tag{1}$$