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Smoothed particle hydrodynamics for coarse-grained modeling of rapid granular flow

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ABSTRACT

We simulated rapid flow in transient plane Couette flows of granular particles using the smoothed particle hydrodynamics (SPH) solutions of a set of continuum equations. This simulation was performed to test the viability of SPH in solving the equations for the solid phase of the two-fluid model associated with fluidization. We found that SPH requires the handling of fewer particles in simulating the collective behavior of rapid granular flow, thereby bolstering expectations of solving the equations for the solid phase in the two-fluid modeling of fluidization. Further work is needed to investigate the effect of terms describing pressure and viscous stress of solids on stability in simulations.

Introduction

In the two-fluid model (TFM) of gas–solid fluidization, the collective flow behavior of discrete particles is treated as a continuum with a stress tensor, which includes the solids pressure and the solids viscous stress, used to close the momentum effects of interparticle collisions and fluctuating motions. To solve the TFM equations, various grid-based algorithms have been developed, and, in recent years, meshless, particle-based methods have also been receiving wide interest. Smoothed particle hydrodynamics (SPH) is representative of these approaches (Monaghan, 1997; Monaghan & Kocharyan, 1995; Xiong, Deng, Wang, & Ge, 2011).

As a particle-based method, SPH was initially developed to simulate astrophysical phenomena (Gingold & Monaghan, 1977; Lucy, 1977). The continuum domain is discretized into a set of Lagrangian particles carrying their own time-constant mass m and time-evolving field properties, such as position $\mathbf{r}$, velocity $\mathbf{v}$, and density ρ. The standard SPH algorithm is explicit and reduces the partial differential equations (PDEs) of the continuum model to a set of ordinary differential equations, being in essence Newton's second law of motion for each SPH particle. Because the number of SPH particles is normally comparable with the number of grid points used in numerical schemes for continuum models, and much less than the number of real particles, the SPH can be viewed as a coarse-graining method if the continuum represents the collective behavior of discrete particles, as is the case for the equations of the solid phase for TFM. Thus, the SPH solution of the TFM can be expected to greatly reduce the computational load. Similar advantages can also be found for the multiphase particle-in-cell (MP-PIC) method (Andrews & O'Rourke, 1996; Li, Song, Benyahia, Wang, & Li, 2012; Snider, 2001) or other parcel-based methods (Bierwisch, Kraft, Riedel, & Moseler, 2009). For this reason, we have adopted the SPH for the TFM simulation of liquid–solid sedimentation, gas–solid bubbling fluidization (Xiong et al., 2011), and recently, gas–solid riser flow (Deng, Liu, Wang, Ge, & Li, 2013).

The fluidized particles move in a similar manner to particles in rapid granular flow or elastic granular flow with varying volume fraction of solids (Campbell, 1990; Goldhirsch, 2003). Solid stress closure largely determines the accuracy of coarse graining and hence needs careful examination. Our previous attempts at SPH simulations of fluidization involve drag (Deng et al., 2013; Xiong et al., 2011), which may dominate the two-phase flow and thus blur the effects of stress closure. Therefore, in this study, we chose rapid granular flow, or granular gas (Campbell, 1990; Goldhirsch, 2003; Goldhirsch & Zanetti, 1993), to validate SPH as a coarse-graining approach for the solid phase. Granular Couette flow is

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first simulated by solving the equations of TFM for the solid phase. Comparison with analytical results is also provided.

SPH model

When the collective behavior of discrete particles is treated as a continuum, the volume fraction of the solid phase has to be included. As in TFM, the continuum equations take the following form except without the inclusion of drag,

$$\frac{\partial\left(\rho_s\theta_s\right)}{\partial t}+\nabla\cdot\left(\rho_s\theta_s\mathbf{v}_s\right)=0, \tag{1}$$

$$\frac{\partial\left(\rho_s\theta_s\mathbf{v}_s\right)}{\partial t}+\nabla\cdot\left(\rho_s\theta_s\mathbf{v}_s\mathbf{v}_s\right)=-\nabla p_s+\mu_s\theta_s\nabla^2\mathbf{v}_s+\rho_s\theta_s\mathbf{g}. \tag{2}$$

The density $\bar{\rho}_i$ (=$\rho_i\theta_i$) of the i-th SPH particle can be computed directly by discretizing the continuity Eq. (1) or by summing the mass of neighboring particles j weighted by a smoothing kernel W, i.e.,

$$\bar{\rho}_i=\sum_j m_j W_{ij}. \tag{3}$$

Actually, the density summation in Eq. (3) is an exact time-independent solution to the SPH continuity equation (Price, 2012). A cubic spline function (Liu & Liu, 2003) is adopted as a smoothing kernel:

$$W\left(\mathbf{r}-\mathbf{r}',h\right)=\alpha_d\begin{cases}4-6R^2+3R^3, & 0\le R<1\\ (2-R)^3, & 1\le R<2\end{cases}, \tag{4}$$

where $R=|\mathbf{r}-\mathbf{r}'|/h$, h is smoothing length, and α_d equals $1/(6h)$, $5/(14\pi h^2)$, and $1/(4\pi h^3)$ in one-, two-, and three-dimensional spaces, respectively. The momentum equation Eq. (2) can be finally reformulated in SPH form, as in recent literature (Deng et al., 2013; Xiong et al., 2011); it reads

$$\frac{d\mathbf{v}_i}{dt}=-\frac{1}{\rho_s^2}\sum_j m_j G\left(\frac{\theta_i+\theta_j}{2}\right)\left(\frac{1}{\theta_i}+\frac{1}{\theta_j}\right)\left(\frac{\mathbf{r}_{ij}}{r_{ij}}\frac{\partial W_{ij}}{\partial r_{ij}}\right)+\mu_i\sum_j m_j\mathbf{v}_{ij}\left(\frac{\theta_i+\theta_j}{\bar{\rho}_i\bar{\rho}_j}\right)\left(\frac{1}{r_{ij}}\frac{\partial W_{ij}}{\partial r_{ij}}\right)+\mathbf{g}, \tag{5}$$

where $\mathbf{r}_{ij}=\mathbf{r}_i-\mathbf{r}_j$. The solids pressure is related to the solid volume fraction θ_s via the elastic modulus, e.g., $\nabla p_s=G\left(\theta_s\right)\nabla\theta_s$ (Gidaspow, 1994), and the solids viscosity coefficient is a constant or is proportional to the solid concentration (Sun & Gidaspow, 1999; Tsuo & Gidaspow, 1990; Yang, Wang, Ge, & Li, 2003). An alternative formulation is the kinetic theory of granular flows (KTGF), which is, however, much more complicated to implement in SPH. Finally, the elastic modulus $G(\theta_s)=10^{8.76\theta_s-1.183}$ used in Jayaswal (1991) is adopted in this work.

To integrate Eqs. (3) and (5), let $\mathbf{R}_i$ denote all the terms on the right-hand side of Eq. (5). With the current values for velocity $\mathbf{v}_i^n$, position $\mathbf{r}_i^n$ and solid density $\bar{\rho}_i^n$ from Eq. (3), the new position $\mathbf{r}_i^{n+1}$ and velocity $\mathbf{v}_i^{n+1}$ for particle i are updated using

$$\mathbf{r}_i^{n+1}=\mathbf{r}_i^n+\Delta t\mathbf{v}_i^n+0.5\Delta t^2\mathbf{R}_i^n, \tag{6}$$

$$\mathbf{v}_i^{n+1}=\mathbf{v}_i^n+\Delta t\mathbf{R}_i^n. \tag{7}$$

Simulation of transient plane Couette flow

The transient plane Couette flow involves a fluid flow (without gravitational effect) between two infinite plates, which are initially stationary and vertically placed at $x=0$ and $x=L$ (see Fig. 1). Once the right plate moves at a constant velocity V_w, the vertical velocity field between the two plates evolves as (Papanastasiou, Georgiou, & Alexandrou, 2000).

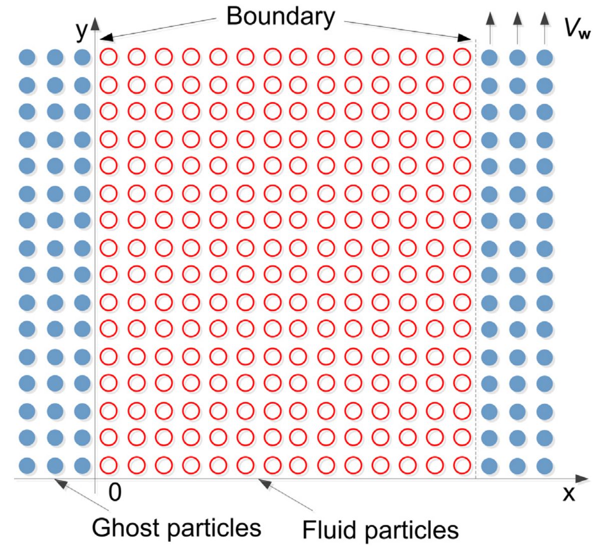

Fig. 1. Initial distribution of SPH particles for transient plane Couette flow.

$$v_y(x,t)=\frac{V_w}{L}x+\sum_{n=0}^{\infty}\frac{2V_w}{n\pi}(-1)^n\sin\left(\frac{n\pi}{L}x\right)\exp\left(-\frac{n^2\pi^2\nu}{L^2}t\right), \tag{8}$$

where $\nu=\mu/\rho$ is the kinematic viscosity of the fluid. The viability of SPH for such flow for a pure fluid has been reported in the literature (Morris, Fox, & Zhu, 1997). For rapid granular flow with density $\theta_s\rho_s$ and dynamic viscosity $\theta_s\mu_s$, we could expect the flow to behave like a fluid with kinematic viscosity μ_s/ρ_s, provided that the stress terms were correctly closed. The fluid property of the rapid granular flow is what we want to validate in this work because the contribution from solids in TFM can be well represented by rapid granular flow except near the close packing state. Table 1 lists the physical parameter values and simulation settings. Using $Re=V_wL/\nu$, the Reynolds number of the flow is 57.14; Fig. 1 shows the initial distribution of particles. The no-slip boundary condition is approximated by placing "ghost particles" outside the plates; for details refer to Morris et al. (1997).

Interparticle distance

The SPH can be viewed as a Lagrangian description of a continuum model. Hence, the number of SPH particles is comparable with the number of points in the Eulerian grids used in solving the continuum model. In practice, this is its main advantage over direct tracking of all physical particles as in the discrete element

Table 1
Physical parameter values and simulation settings.

Parameter	Value
Distant between two plates, L (m)	0.01
Solid particle density, ρ_s (kg/m^3)	2000
Solid viscosity, μ_s (Pa·s)	$k\theta_s$
Solid particle diameter, d_s (m)	3.0×10^{-4}
Initial solids volume fraction, θ_{s0}	0.35
Velocity of wall, V_w (m/s)	1.0
Kernel function, $W(\lvert\mathbf{r}_{ij}\rvert, h)$	Cubic spline
Smoothing length, h (m)	2.5×10^{-4}
Time step, Δt (s)	1.0×10^{-5}
SPH particle number in x direction, $N_{SPH,x}$	50