Engineering Structures 173 (2018) 269–282

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Analytical solution of three-dimensional two-layer composite beam with interlayer slips

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ARTICLE INFO

Keywords:
Analytical solution
Three-dimensional
Composite
Interlayer slip
Beam
Benchmark

ABSTRACT

This paper focuses on development of a new mathematical model and its analytical solution for the analysis of the mechanical behavior of geometrically and materially linear three-dimensional (3D) two-layer composite beams with interlayer slips between the layers. Consequently, the analytical solution of the mechanical behavior of elastic 3D two-layer composite beams with interlayer slips is derived for the first time. In the illustrative example, the cantilever 3D two-layer timber-concrete composite beam with interlayer slips is analysed, and the analytical results for various kinematic and equilibrium quantities are given. Besides, it is demonstrated that in case of the shear-stiff 3D two-layer composite beams with interlayer slips, the deformations in transverse *XZ* and lateral *XY* planes are mutually independent. The results obtained can be used as a benchmark solution.

1. Introduction

In the past few decades, the layered composite structures have been used increasingly in many structural applications. Nowadays, they are widely used in aerospace, automotive, naval, microelectronics, underwater, nuclear, biomedical, mechanical, civil, and many other engineering problems. Due to their attractive properties, like high strength-to-weight and stiffness-to-weight ratios, corrosion resistance, design flexibility, durability, fire protection, thermal insulation, dissipation, etc., the application of layered composite structures in civil engineering problems is increased tremendously. Typical members in most common civil applications such as building, bridge, and shelter construction, are steel-concrete composite beams, layered wooden beams, wood-concrete and timber-steel floor systems, sandwich and glass laminated structures, reinforced concrete beams strengthened by adhesively bonded steel plates or fiber-reinforced polymers, and so on.

The overall mechanical performance of these structures depends not only on material properties of adjacent layers, but also to a large degree on the type of the connection between them. Usually, the layers of different materials are attached to each other by discrete mechanical devices, e.g. shear studs, screws, nails, bolts, or continuously by elastomeric or viscoelastic adhesive joints. However, in practice, these connectors are deformable with finite stiffness. As a result, a relative displacement between the layers can occur, which is called an interlayer slip. If it has a sufficient magnitude, it can affect the mechanical behavior of layered composite beams significantly. Therefore, the investigation of the effects of the interlayer slip on the performance of layered composite beams is of great importance.

Consequently, over the years, the effect of the interlayer slip on mechanical performance of layered structures has attracted much research attention. Hence, a large number of studies exist in literature. The one of the earliest theory on the interlayer slip was the analytical model proposed by Newmark et al. [1], who analysed a two-layer steel-concrete beam with deformable shear connectors by linear Euler-Bernoulli beam theory. Following this work, many interesting analytical models have been developed for studying layered composite beams with interlayer slip, considering two-layers [2–8], three- or more layers [9–12], Timoshenko two-layer beam [13–15], time-dependent effects [16], higher order theories [17], non-linear or bi-linear interface laws [18,19], 2-D elasticity [20], uplift [21,22], buckling [23,24], dynamics [25], curved layered beams [26], and so forth. Furthermore, the innovative analytical modelling and exact solutions have also been presented of three-layered sandwich and glass plates with compliant inner interlayer by Foraboschi [27–29].

On the other hand, besides these analytical works, several numerical models for investigation the performance of layered composite beams with interlayer slip have been developed as well. Therefore, a great number of papers referring to finite elements [30–42], dynamics and vibration [43–45], steel-concrete and reinforced concrete beams [46–49], fire [50,51], and so on, have been published recently.

However, as far as the authors' awareness, the only analytical solutions of the mechanical behavior of three-dimensional (3D) two-layer

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https://doi.org/10.1016/j.engstruct.2018.06.108
Received 26 January 2018; Received in revised form 21 June 2018; Accepted 27 June 2018
0141-0296/

Nomenclature

A	cross-sectional area (cm^2)
b_c	contact width (cm)
E	Young's modulus (kN/cm^2)
G	shear modulus (kN/cm^2)
I_X	torsional moment of inertia around X axis (cm^4)
I_Y, I_Z	bending moments of inertia around Y and Z axes (cm^4)
I_{YZ}	centrifugal moment of inertia (cm^4)
K_X, K_Y	contact stiffness in X and Y directions (kN/cm^2)
L	beam length (cm)
$\mathscr{M}_X$, $\mathscr{M}_Y$, $\mathscr{M}_Z$	X, Z, Y components of a distributed moment load (kN cm/cm)
M_X	torsional moment (kN cm)
M_Y, M_Z	bending moment components in Y and Z directions (kN cm)
N_X	axial force (kN)
N_Y, N_Z	shear force components in Y and Z directions (kN)
p_X, p_Y, p_Z	contact tractions in X, Y, Z directions (kN/cm)
$\mathscr{P}_X$, $\mathscr{P}_Y$, $\mathscr{P}_Z$	X, Y, Z components of a distributed load (kN/cm)
S_Y, S_Z	static moments around Y and Z axes (cm^3)
u	axial displacement (cm)
v, w	transverse displacements in Y and Z directions (cm)

Greek letters

γ_Y, γ_Z	shear strains
Δu, Δv, Δw	relative displacements in X, Y, and Z directions (cm)
$\Delta\omega_X$, $\Delta\omega_Y$, $\Delta\omega_Z$	relative rotations around X, Y, and Z axes (rad)
Δ_X, Δ_Y	slips in X and Y directions (cm)
Δ_Z	uplift in Z direction (cm)
ε_{XX}	extensional strain
κ_X	torsional strain (rad/cm)
κ_Y, κ_Z	bending strains (rad/cm)
ω_X	torsional rotation (rad)
ω_Y, ω_Z	rotations around Y and Z axes (rad)

Subscripts and superscripts

ex	external
i	layer or material
c	contact

composite beams with interlayer slips available in the existing literature are the ones proposed by Challamel and Girhammar [52,53]. They derived analytical solutions for the lateral-torsional buckling problem of partially horizontally and vertically layered or sandwich-type beams under uniform moment. Nevertheless, to date, no analytical solution has been derived yet for general bending problem of 3D two-layer composite beams with interlayer slips under general loading.

Thus, the main purpose of this paper is to fill this gap and derive a novel mathematical model and its analytical solution for the investigation of mechanical flexural behavior of 3D two-layer composite beams with interlayer slips subjected to general loading conditions. Accordingly, this novel analytical formulation will definitely contribute importantly to the knowledge about the behavior of the 3D layered composite beams with interlayer slips between adjacent layers.

Finally, two illustrative examples are given to present the analytical solution of the 3D two-layer composite beams with interlayer slip. In the first example the cantilever timber-concrete 3D composite beam with interlayer slips is analysed which is subjected to lateral and transverse eccentric point loads. Then, in the second example, a simultaneously simply supported and cantilever 3D two-layer timber-timber with uniformly distributed load is analysed. In both cases the analytical solutions of the problem are derived and presented in graphical and tabulated form.

2. Problem foundations

2.1. Preliminaries

Consider an initially straight, three-dimensional (3D), two-layer composite beam of undeformed length L as shown in Fig. 1. Layers, which can in general be made from different materials, are marked by letters a and b, respectively. They have equal undeformed lengths, i.e. $L^a \equiv L^b$, and arbitrary prismatic cross-sections, A_x^a and A_x^b. The layers are joined by an interface that has negligible thickness and finite stiffness. The two-layer 3D composite beam is placed in a 3D cartesian coordinate system with spatial coordinates (X, Y, Z) and unit base vectors $\mathbf{E}_X$, $\mathbf{E}_Y$, $\mathbf{E}_Z$. The undeformed reference axis of the two-layer 3D composite beam is common to both layers. It lies along the global

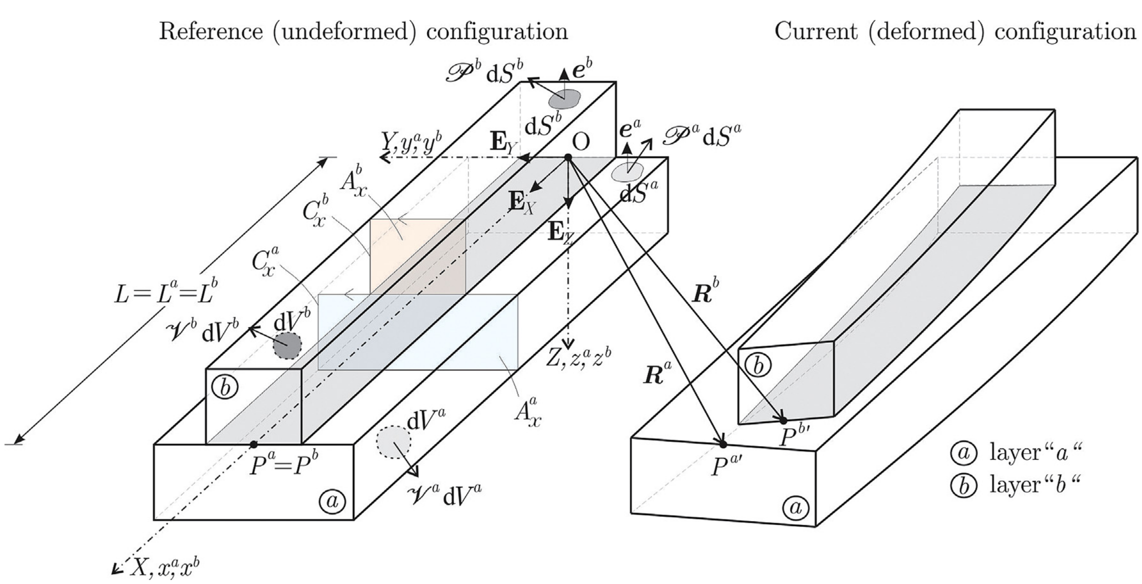

Fig. 1. Shematic of the two-layer three-dimensional composite beam with interfacial slips. Reference (undeformed) and current (deformed) configurations.