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An integrated approach for predicting the shear capacity of fibre reinforced concrete beams

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ABSTRACT

This paper describes the development of an integrated design approach for determining shear capacity of flexurally reinforced steel fibre reinforced concrete members. The approach considers fibre distribution profile, fibre pull-out resistance and the modified compression field theory integrated using a comprehensive strategy. To assess the performance of the developed model, a database consisting of 122 steel fibre reinforced and prestressed concrete beams failing in shear was assembled from available literature. The model predictions were shown to correlate well with the test data. The performance of the analytical model was also compared to predictions attained by the two approaches recommended by the *fib* Model Code 2010, one based on an empirical equation and the other on the modified compression field theory approach. The predictive performance of the proposed approach was also assessed by using the *Demerit Points Classification* (*DPC*), being the prediction as better as lower is the total penalty points provided by the classification. The model developed in this paper demonstrated a superior performance to those of the Model Code, with a higher predictive performance in terms of safety and reliability.

1. Introduction

The first fibre reinforced beams tested for shear were those of Batson et al. [1]. They investigated a range of fibre types and geometries, as well as span-to-depth ratios. Since this time, numerous studies have demonstrated that the presence of steel fibres increases the shear strength of concrete beams [2–26]. Even though the costs steel fibres may exceed substantially that of relatively cheap steel ligatures for the carrying of shear stress resultants, there is potential for significant savings in site labour costs. Whether or not fibres can replace conventional transverse steel bar reinforcement in reinforced concrete beams is a matter that needs to be addressed through analysis of experimental data and in models development.

For determining the reliability of various competing design models, a database of reliable experimental tests must first be established. One of early documented studies with an extensive data collection is that of Adebar et al. [26]. Their study identified 413 SFRC beams reported in the literature as being tested in shear, although many of the tests were limited by their flexural strength. Recent articles have been dedicated to prepare substantive test databases in this subject [27–30].

It is well recognized that the post-cracking tensile, or pull-out, response of fibre reinforcement embedded in cement based materials is the distinguishing characteristic defining performance in terms of serviceability (including stiffness), durability and strength of fibre-reinforced structural elements. This is represented by the stress-crack width relationship, σ–w. In structures governed by shear, fibre reinforcement increases the stiffness and strength of the shear stress transfer across cracks [31]; however, a methodology to capture the contribution of fibre reinforcement to shear strength enhancement is challenging.

Despite the high potential, a consensual modelling approach does not yet exist for predicting shear strength of fibre reinforced concrete (FRC) beams at high accuracy, where flexure is resisted primarily by bar reinforcement or tendons. While the models developed in this study are generic in their nature, and apply across the breadth of fibres produced of different materials, experimental testing to date of reinforced concrete FRC beams has almost exclusively been undertaken using steel fibres; these are herein termed R/SFRC beams.

Two approaches for the determination of the shear strength of R/SFRC are described in the *fib* Model Code 2010 [31]; the first has its basis in a Eurocode 2 empirical design strategy [6], the second using a philosophy founded from the modified compression field theory

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(MCFT) [27]. In this paper a physical-mechanical model is developed for assessing the strength of R/SFRC beams failing in shear. The approach integrates fibre orientation profile along the critical diagonal crack (CDC), the relevant pull-out mechanisms of steel fibres and the fundamental concepts of the MCFT. The models are compared to test data of 122 beams collected from the literature, and the results are reported herein.

2. MC2010 approaches for shear strength of R/SFRC beams

2.1. Introduction

The *fib* Model Code 2010 [31] outlines two approaches for determining the shear capacity of R/SFRC beams. The first is based on a modification to the Eurocode model and the second is founded on the MCFT. The backgrounds of these two models are described briefly in this section.

2.2. Approach based on the concept of residual flexural strength for FRC

This approach, denoted in this paper as MC2010-EEN, is based on the empirical equation developed in [6]. By this approach, the shear resistance is obtained from [32]:

$$V_{Rd} = V_{Rd,F} + V_{Rd,s} \tag{1}$$

where $V_{Rd,F}$ and $V_{Rd,s}$ are the components of shear carried by fibres and shear ligatures, respectively. The fibres component is given by:

$$V_{Rd,F} = \left\{\frac{0.18}{\gamma_c}k\left[100\rho_{eq}\left(1 + 7.5\frac{f_{Ftuk}}{f_{ctk}}\right)f_{ck}\right]^{1/3} + 0.15\sigma_{cp}\right\}b_w d_{eq} \tag{2a}$$

where

$$\rho_{eq} = \frac{A_l}{b_w d_l} + \frac{A_p}{b_w d_p} \tag{2b}$$

is the flexural reinforcement ratio in the general case of a R/SFRC beam with passive and prestressed reinforcements, being A_l and d_l, and A_p and d_p their corresponding cross sectional area and internal arm, and b_w the width of the web of the section, while

$$d_{eq} = \frac{d_l A_l + d_p A_p}{A_l + A_p} \tag{2c}$$

is the equivalent internal arm of the flexural reinforcement.

In Eq. (2a) f_{Ftuk} is the post-cracking residual tensile strength obtained from either a direct tensile test of by inverse analysis on prism bending test data; f_{ctk} is the characteristic tensile strength of the FRC; γ_c is a partial safety factor ($\gamma_c = 1.5$), $\sigma_{cp} = N_{sd}/A_c < 0.2f_{ck}/\gamma_c$ is the average stress acting on the concrete cross section, A_c, for an axial force, N_{sd}, due to loading or prestressing actions ($N_{sd} > 0$ for compression); and k is a factor that takes into account the size effect and given by:

$$k = 1 + \sqrt{200/d_{eq}} \leqslant 2.0 \ (d_{eq} \text{ in mm}) \tag{3}$$

The characteristic post-cracking residual tensile strength (f_{Ftuk}) of the SFRC for shear is determined at a crack opening displacement (COD) of $w_u = 1.5$ mm, and is given by:

$$f_{Ftuk} = 0.45f_{R1k} - 0.6(0.65f_{R1k} - 0.5f_{R3k}) \geqslant 0 \tag{4}$$

where f_{R1k} and f_{R3k} are flexural strengths determined in accordance with MC2010. In the database that will be introduced in Section 4 for the assessment of the predictive performance of the MC2010 approaches, the f_{Ri} values of the SFRC of some beams are not available. For these cases, the f_{R1k} and f_{R3k} are estimated from the relationship proposed by Moraes-Neto [33] and Moraes-Neto et al. [34]:

$$f_{Ri} = k_1(V_f l_f/d_f)^{k_2} \ i = 1,\ 2,\ 3,\ 4 \tag{5}$$

where k_1 = 10.5, 9.2, 8.0, 7.0 and k_2 = 0.80, 0.75, 0.70, 0.65 for f_{R1}, f_{R2}, f_{R3} and f_{R4}, respectively (the values for f_{R2} are interpolated from those for f_{R1} and f_{R3}). Although the authors recognize that f_{Ri} values are not only dependent on the V_f, l_f and d_f fibre characteristics, later it will be demonstrated they can be predicted with reasonable accuracy from Eq. (5) in the context of this study.

The equation for determining the contribution of the transverse bar reinforcement ($V_{Rd,s}$) is not provided here since the present database does not include any R/SFRC beams with this reinforcement, but it can be found in the MC2010.

The design shear resistance cannot be greater than the crushing capacity of concrete in the web:

$$V_{Rd,\max} = k_c\frac{f_{ck}}{\gamma_c}b_w z\frac{\cot\theta + \cot\alpha}{1 + \cot^2\theta} \tag{6}$$

where $z = 0.9d_{eq}$ is the effective shear depth, θ is the inclination of the CDC, $k_c = k_\varepsilon\eta_{fc}$, $k_\varepsilon = 0.55$ and:

$$\eta_{fc} = (30/f_{ck})^{1/3} \leqslant 1.0 \ (f_{ck} \text{ in MPa}) \tag{7}$$

2.3. 3Approach based on the modified compression field theory (MCFT)

The second approach proposed in the MC2010 for the determination of the shear capacity of R/SFRC beams was developed from the MCFT [27], and is herein denoted as MC2010_MCFT. By this approach the shear capacity of an R/SFRC beam is calculated from Eq. (1) with:

$$V_{Rd,F} = \frac{1}{\gamma_F}(k_v\sqrt{f_{ck}} + k_f\, f_{Ftuk}(w)\cot\theta)zb_w \ldots \text{ with } \sqrt{f_{ck}} \leqslant 8 \text{ MPa} \tag{8}$$

where $k_f = 0.8$ is factor to account for fibre dispersion, f_{Ftuk} is the post-cracking residual tensile strength obtained from a direct tensile test, and k_v is a size effect parameter.

The size/strain effect parameter is related to the longitudinal strain determined at the mid-depth of the section (ε_x) and to the size of the largest aggregate particles (d_g) by:

$$k_v = \begin{cases} \dfrac{0.4}{1+1500\varepsilon_x}\dfrac{1300}{1000+zk_{dg}} & \ldots \text{ for } \rho_w < 0.08\sqrt{f_{ck}}/f_{lyk} \\ \dfrac{0.4}{1+1500\varepsilon_x} & \ldots \text{ for } \rho_w \geqslant 0.08\sqrt{f_{ck}}/f_{lyk} \end{cases} > 0.0 \tag{9a}$$

$$k_{dg} = \begin{cases} \dfrac{32}{16+d_g} \geqslant 0.75 & \ldots \text{ for normal–weight concrete with } f_{ck} \leqslant 70\text{MPa} \\ 2.0 & \ldots \text{ for } f_{ck} > 70 \text{ MPa and for light–weight concrete} \end{cases} \tag{9b}$$

where f_{lyk} is the characteristic value of the yield strength of the main longitudinal bars, and d_g is in mm.

The mid-depth longitudinal strain is calculated for reinforced concrete (RC) and prestressed concrete (PC) beams from:

$$\varepsilon_x = \begin{cases} \dfrac{1}{2E_l A_l}\left(\dfrac{M_{Ed}}{z} + \dfrac{V_{Ed}\cot\theta}{2} + N_{Ed}\left(\dfrac{1}{2}-\dfrac{\Delta e}{z}\right)\right) & \ldots \text{ for RC beams} \\ \dfrac{\left(\dfrac{M_{Ed}}{z} + \dfrac{V_{Ed}\cot\theta}{2} + N_{Ed}\left(\dfrac{1}{2}-\dfrac{\Delta e}{z}\right)\right)}{2\left(\dfrac{z_l}{z}E_l A_l + \dfrac{z_p}{z}E_p A_p\right)} & \ldots \text{ for PC beams} \end{cases} \tag{10}$$

within the limits $0 \le \varepsilon_x \le 0.003$.

In Eq. (10), M_{Ed}, V_{Ed} and N_{Ed} are the design values of the bending moment, shear and axial forces acting on the cross section, respectively. The bending moment and shear force are taken as positive; the axial force is positive for tension and negative for compression. The eccentricity of the beam axis with respect to section mid-depth (Δe), shown in Fig. 1, is a positive value when positioned above the centre of gravity of the cross section. For R/SFRC hybrid flexurally reinforced beams, with passive (subscript "l") and prestressed (subscript "p") reinforcements, the effective shear depth, z, is evaluated from: