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Analytical derivation of the effective creep coefficients for timber-concrete composite structures

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ABSTRACT

The limitation of the deflection in the long term is often a governing condition when designing timber-concrete composite structures. In order to calculate the long term deflection, the creep deformation is often described by means of creep coefficients. These coefficients are the ratio between the creep and the elastic strain. However in a timber-concrete composite structure the elastic strain is not constant over time, hence an effective creep coefficient has to be introduced and used. This coefficient differs from the pure material creep coefficient provided in the standards. This paper presents an analytical derivation of the effective creep coefficients for a timber-concrete composite structure. Based on the results of an extensive parametric study, a simplified closed-form solution is proposed for the calculation of the effective creep coefficients.

1. Introduction

The design of timber-concrete composite structures is often governed by the limitation of the deflection. This deflection is strongly influenced by the time-dependent behaviour of the components, namely creep (including mechano-sorptive creep) and shrinkage of timber and concrete as well as creep (and mechano-sorptive creep) of the connection.

In order to describe the long-term behaviour, several numerical models have been developed (see among others [1–10]). In these models, the rheological behaviour of timber, concrete and connection are linked to each other by means of relationships, describing the effect of the composite action with respect to the deformability of the connection (see Fig. 1).

These models were checked against experimental measurements, showing maximum differences between the predicted deformation and the measured deformation of about 10% and therefore acceptable accuracy (see among others [1,2,11]). As next step towards deriving simplified design rules, extensive parametric studies can be performed with these models, evaluating the effects in the long-term. Unfortunately, no simplified approach can be derived, since the re-evaluated creep coefficients vary significantly depending on several variables (see Figs. 2 and 3). As shown in Fig. 2, the global creep coefficient of timber-concrete composite structures defined as the ratio of the creep and the elastic deformation can reach values between 1.0 and 4.0. This large variability of results is obviously not accurate enough for the design purposes.

If the creep coefficients of the single components are re-evaluated from the results of the numerical simulations, significant differences with the material creep coefficient of concrete and the re-evaluated creep coefficient obtained from the numerical model exist (see Fig. 3). So the commonly used pure material creep coefficients do not lead to accurate results for the evaluation of the long-term behaviour of timber-concrete composite structures.

The reason for these differences is that the creep coefficient is defined as the ratio of the creep strain and the current elastic strain. In a statically determined system, the stresses are constant over the time (pure creep problem), and the long term strain can be calculated as

$$\varepsilon(t) = \varepsilon_{elastic} + \varepsilon_{creep} = \varepsilon_{elastic}\cdot(1 + k_{def}) \tag{1}$$

where k_{def} signifies the creep coefficient of timber as defined in [13].

The final deformation can be calculated using an effective Modulus of Elasticity.

$$E_{eff} = \frac{E}{1 + k_{def}} \tag{2}$$

If the system cannot deform over time, the problem of assessing the time dependent behaviour is defined as relaxation. In this case, the resulting deformation is constant over time

$$\frac{d}{dt}\varepsilon(t) = \frac{d}{dt}\varepsilon_{elastic} + \frac{d}{dt}\varepsilon_{creep} = 0 \tag{3}$$

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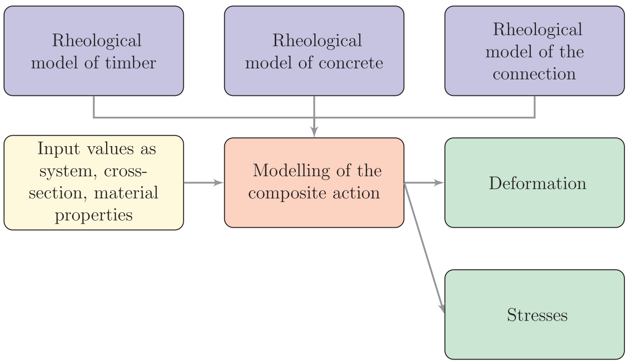


Fig. 1. Flow diagram of the models describing the long-term behaviour of timber-concrete composite systems.

[14] transformed this equation into following expression:

$$\frac{1}{E}\cdot\frac{d}{dt}\sigma(t) + \frac{\sigma}{E}\cdot\frac{d}{dt}k_{def}(t) = 0 \tag{4}$$

The solution of this differential equation is given by following equation:

$$\sigma = \sigma_0\cdot e^{-k_{def}(t)} \tag{5}$$

This equation can also be described via the use of an effective Modulus of Elasticity, which differs from the effective Modulus of Elasticity of a pure creep behaviour as given in Eq. (2).

$$E_{eff} = \frac{E}{e^{k_{def}(t)}} = \frac{E}{1 + \underbrace{\varphi_{composite}}_{e^{k_{def}(t)}-1}} \tag{6}$$

A timber-concrete composite system is an internally statically indetermined system due to the presence of a (flexible) connection, where the single components cannot move freely. Therefore the long-term behaviour cannot be explained as a pure creep phenomenon. On the other hand the components are deformable and therefore stresses are redistributed, so the long-term behaviour cannot classified as a pure relaxation phenomenon either. Therefore the effective creep coefficient in a composite structure $\varphi_{composite}$ is inbetween the creep coefficient for pure creep and the creep coefficient for relaxation (see Fig. 4).

$$k_{def} \leqslant \varphi_{composite} \leqslant e^{k_{def}}-1 \tag{7}$$

For the design of timber-concrete composite structures this difference cannot be neglected since the limitation of the deflection is often the governing condition. Therefore the evaluation of the creep coefficients in the composite systems should be as accurate as possible.

2. Analytical approach for the determination of effective creep coefficients in composite systems

The deformation caused by creep is often described by the creep coefficient, which is the ratio between the creep and the elastic strain

$$\varphi = \frac{\varepsilon_{\text{creep}}}{\varepsilon_{\text{elastic}}} = \frac{\Delta l_{\text{creep}}}{\Delta l_{\text{elastic}}} = \frac{\Delta l_{\text{total}}-\Delta l_{\text{elastic}}}{\Delta l_{\text{elastic}}} \tag{8}$$

where φ creep coefficient
Δl increase in deformation

In a statically indetermined system, the distribution of the internal forces depends on the stiffness of the single components as well as the creep strain caused by the internal stresses. Therefore the elastic strain $\varepsilon_{\text{elastic}}$ is not constant over time.

One possible way to determine the internal forces in a statically indetermined system is to cut the system at certain locations and replace the effect to the other part of the system via a coupling force $X(t)$. The coupling force $X(t)$ is determined by superimposing the deformation due to the external load and the coupling force in order to set the resulting deformation at the location of the section cut to zero.

If this concept is applied to the long-term scenario, the total deflection can be expressed as the sum of the elastic and the time dependent deflection:

$$\Delta l_{\text{total},o} = \underbrace{\delta_{o,1,1}\cdot X_p(t)}_{\Delta l_{elastic}} + \underbrace{\int_0^{\varphi_{o,\text{M}}} \delta_{o,1,1}\cdot X_p(t) d\varphi_{o,\text{M}}}_{\Delta l_{creep}} \tag{9}$$

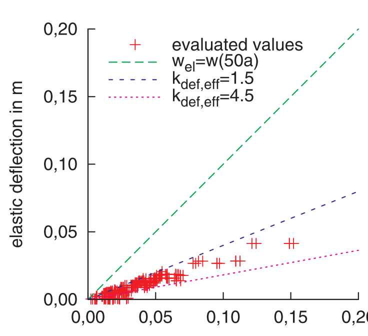
where o, p $= u, w$ and $o \neq p$
u component u i.e. timber
w component w i.e. concrete
$\varphi_{o,\text{M}}$ material creep coefficient of component o e.g. according to the standard of the material
$X_p(t)$ coupling force between the composite members

The composite creep coefficient $\varphi_{o,\text{C}}$ can be determined with the assumption of a Modulus of Elasticity constant over time by following expression.

$$\varphi_{o,\text{C}} = \frac{\int_0^{\varphi_{o,\text{M}}} \delta_{o,1,1}\cdot X_p(t) d\varphi_{o,\text{M}}}{\delta_{o,1,1}\cdot X_p(t)} = \frac{\int_0^{\varphi_{o,\text{M}}} X_p(t) d\varphi_{o,\text{M}}}{X_p(t)} \tag{10}$$

In composite structures the shear forces are transferred continuously along the beam axis. It is assumed that the slip distribution along the beam axis over time is affine to the initial slip distribution. Therefore the shear flow in the shear connection between timber and concrete $q(x, t)$ can be determined by (see Fig. 5):

$$q(x, t) = f(x)\cdot X(t) \tag{11}$$



- massive timber-concrete composite decks
  - $b_{timber} = b_{concrete}$
  - $h_{timber} : h_{concrete}$ =1:1 to 3:1
- effective creep coefficient

$$k_{def,eff} = \frac{w(t=50y)-1}{w_{elastic}}$$

Fig. 2. Range of the global creep coefficient $k_{def,eff}$ for the evaluation of the final deflection.