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Effective Stresses in Multiphasic Porous Media: A thermodynamic investigation of a fully non-linear model with compressible constituents

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HIGHLIGHTS

- Effective stresses in unsaturated soil.
- Continuum-mechanical and thermodynamical investigation of a triphasic medium of solid, pore liquid and pore gas.
- Investigation of the full set of constitutive equations.
- Split of volumetric deformations of the pore space and the basic solid material.
- Nonlinear elastic solid deformations.

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ABSTRACT

In literature, it is often said that the "Principle of Effective Stresses" has been introduced in soil mechanics by Karl von Terzaghi in 1925 in his famous book "Erdbaumechanik auf bodenphysikalischer Grundlage" stating that the total stress acting in a soil body can be split into an effective stress and the pore-water pressure. While the effective stress is coupled to the soil deformation, the pore-water pressure, in particular its gradient, governs the pore-fluid flow usually following Darcy's law. In earlier days, Terzaghi's principle has been taken as a constitutive equation not only under fully but also under partially saturated conditions, although doubts came up with regard to its overall validity.

Starting with a short historical review, the present article aims at explaining the effective-stress principle from a general point of view on the basis of the Theory of Porous Media and provides an elaborated example for both unsaturated and saturated soil conditions by consideration of compressible and incompressible solid and pore-fluid materials. Based on this approach, it will be proven that Terzaghi's basic assumption of the effective-stress principle is valid, except for materially compressible porous solids.

1. Introduction

The "Principle of Effective Stresses" is a very important ingredient in porous-media mechanics and, especially, in soil mechanical applications, since it indicates that part of the total stress that is governing the strength of the soil, its stress–strain relationship and its stability behaviour. However, although the effective-stress principle has basically been given for fully saturated conditions and materially incompressible pore fluids, it has also freely been taken for other situations, such as partially saturated soil.

Usually, the principle is dated back to 1925 when Karl von Terzaghi published his famous book "Erdbaumechanik auf bodenphysikalischer Grundlage",[1] although Terzaghi mentioned it in an earlier work.[2] A closer look to the literature reveals that the history of the principle is a little bit more complicated and has different actors. In order to give the reader an impression of the basis of the principle, a short part of the article "The development of the concept of effective stresses" published by Reint de Boer and the author in 1990[3] is cited. On p. 83 of the corresponding issue, it is said in the article that

> *according to Skempton,[4] the idea of effective stress was formulated by von Terzaghi in 1936.[5] Skempton stated in a paper with the title "Significance of Terzaghi's Concept of Effective Stress":*

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The principle of effective stress has been stated by Terzaghi in the following terms: "The stresses in any point of a section through a mass of soil can be computed from the total principal stresses σ_1, σ_2, σ_3 *which act in this point. If the voids of the soil are filled with water under a stress u, the total principal stresses consist of two parts. One part, u, acts in the water and in the solid in every direction with equal intensity. It is called the neutral stress (or the pore-water pressure). The balance* $\dot\sigma_1 = \sigma_1 - u$, $\dot\sigma_2 = \sigma_2 - u$ and $\dot\sigma_3 = \sigma_3 - u$ *represents an excess over the neutral stress u and it has its seat exclusively in the solid phase of the soil.*

This fraction of the total principal stresses will be called the effective principal stresses.... A change in the neutral stress u produces practically no volume change and has practically no influence on the stress conditions for failure.... Porous materials (such as sand, clay and concrete) react to a change of u as if they were incompressible and as if their internal friction were equal to zero. All the measurable effects of a change of stress, such as compression, distortion and a change of shearing resistance are exclusively due to changes in the effective stresses $\dot\sigma_1$, $\dot\sigma_2$, *and* $\dot\sigma_3$. *Hence every investigation of the stability of a saturated body of soil requires the knowledge of both the total and the neutral stresses."*

In the article of 1990, *we also concluded that it is difficult to believe that von Terzaghi should be the only one who discovered the idea of effective stress, if one considers Fillunger's total work on the mechanical behaviour of porous media, since Fillunger*[6] *had already stated in his first study on the theory of water-filled porous media that the pore-water pressure has no influence on the strength of the solid skeleton.*

The authors closed their historical remarks on the principle of effective stresses with the statement: *In conclusion, it seems that neither Fillunger*[7] *nor von Terzaghi*[5] *considered the mechanical effect of effective stress to be as important as a general principle. Obviously, this mechanical effect is also developed independently. Nowhere... they referred to this subject.*

In a continuum-mechanical setting, the principle of effective stresses reads

$$\mathbf{T} = \mathbf{T}_{eff} - p\,\mathbf{I}, \tag{1}$$

where $\mathbf{T}$ is the Cauchy stress of the soil body, $\mathbf{T}_{eff}$ the effective stress, p the pore pressure and $\mathbf{I}$ the second-order identity. Note in passing that in contrast to soil mechanics, where the pressure is given a positive value, continuum mechanics always indicated traction forces as positive rather than compression leading to the difference of (1) compared to Terzaghi's definition reading with $\alpha = 1, 2, 3$:

$$\sigma_\alpha = \dot\sigma_\alpha + u. \tag{2}$$

Various other authors included a factor η in the effective-stress expression (1) such that

$$\mathbf{T} = \mathbf{T}_{eff} - \eta\, p\,\mathbf{I} \tag{3}$$

and interpreted η differently. A good overview on these interpretations can be found in the article by Lade and de Boer,[8] cf. Fig. 1. From these considerations, it can be concluded that up to 1973 (Bishop[9]) most of the interpretations of η have been based on experiments, where jacketed and unjacketed specimen have been tested under drained and undrained boundary conditions. As can be seen from Fig. 1, η has either been chosen to 1 or to the porosity n or somehow in between, where either the effective grain contact area a or the compressibility factor K/K_s or its inverse has been used.

So far, the authors cited above did not make an attempt to introduce the stress state in saturated or in partially saturated soil by means of continuum-mechanical investigations including thermodynamic considerations such as fulfilling the second law of thermodynamics or the entropy inequality, respectively. In contrast, younger articles published in 1996 (de Boer and Bluhm[10]), in 2001 (Gray and Schrefler[11]) or in 2006 (Borja[12]) avoided this drawback. While de Boer and Bluhm investigated a compressible solid material with compressible and incompressible pore fluids and described saturated and unsaturated soil material, they came to the final conclusion that the effective-stress principle only holds in case of a materially incompressible solid filled with an incompressible liquid. However, the article remains somehow vague without specified constitutive examples proving the validity of their statements. In Gray and Schrefler,[11] a thermodynamic investigation is carried out for partially saturated soil on the basis of the inclusion of the concept of interfacial areas and *the solid surface fraction in contact with each fluid as the weighting parameter... (rather) than simply the respective degree of saturation.... By* use of this setting, they arrived at complicated expressions that, in a simplified version, yields an extension of Bishop's formula of 1959[13] reading

$$\mathbf{T} = \mathbf{T}_{eff} - [\,\chi\, p^w + (1-\chi)\, p^n\,]\,\mathbf{I}, \tag{4}$$

where p^w and p^n are the effective pressures of the wetting and the non-wetting phases, usually the pore water and the pore air. Considering (4), this is the standard formulation of the effective stress in unsaturated soil, if the Bishop parameter χ is substituted by the saturation s^w of the wetting phase, cf. for example, Hassanizadeh and Gray,[14] Lewis and Schrefler[15] or Ehlers.[16] Apart from these results, Borja[12] succeeded on the basis of the balance equations of mixture theories and by use of some skilful assumptions to obtain

Table 1(a). Expressions for η for stress–strain behaviour of soil, concrete and rock

η	Reference
1	Terzaghi (1923, 1936) Hubbert & Rubey (1959, 1960) Skempton (1960) and many others
n n = porosity	Hoffman (1928) Fillunger (1930) Terzaghi (1945) Lubinski (1954) Biot (1955)
$n \leqslant \eta \leqslant 1$	Schiffman (1970)
$1 - a$ a = effective grain contact area per unit area of plane	Skempton & Bishop (1954) Bishop (1955, 1960) Skempton (1960)
$1 - C_s/C$ or $1 - K/K_s$ $C_s = 1/K_s$ = compressibility of solid material ('grains') $C = 1/K$ = compressibility of skeleton	Biot (1941) Gassmann (1951) Biot & Willis (1957) Geertsma (1957, 1966) Skempton (1960) Serafim (1964) Nur & Byerlee (1971) Bishop (1973)
$1 - (1 - n)C_s/C$	Suklje (1969)

Fig. 1. Table 1 (a) taken from Lade and de Boer.[8]