International Journal of Solids and Structures 000 (2018) 1–23

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Nonlinear dynamic response of acoustically excited and thermally loaded composite plates resting on elastic foundations

Liu Liu[a,*], Jian Ren[a], Ti-Ren He[a], Georgia Kardomateas[b]

[a] School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, PR China
[b] School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150, USA

ARTICLE INFO

Article history:
Revised 31 October 2017
Available online xxx

Keywords:
Thermal protection system
Dynamic response
Strain histogram
Snap-through
Non-Gaussian
Skewness
Geometrical nonlinearity

ABSTRACT

In order to ensure integrity of thermal protection system (TPS) subjected to a combination of thermal and acoustic loadings, a thin composite plate resting on a two-parameter elastic foundation is used to characterize the behavior of the thin top facesheet of TPS. The nonlinear dynamic response of a thermal loaded, acoustic excited plate is investigated. A theoretical model is developed based on Kirchhoff thin plate assumptions and von Kármán-type equation. General static condensation and Galerkin's method are used to derive a set of ordinary differential equations with cubic nonlinearity related to nonlinear coupling between mid-plane stretching and transverse deflection. The reduced-order model has been validated by comparison of postbuckled displacements with those obtained from full-order FEM analysis. Variations of transverse displacement and in-plane strain statistics with acoustic loading level and temperature rising are presented. It is proposed that the in-plane strain located on the plate surface is dominated by the competition of the linear and quadratic nonlinear modal amplitude terms, thus the characteristic of the strain histogram can be used to identify oscillation transition from no snap-through to persistent dynamic snap-through for the thermally buckled plate. The skewness of the strain histogram can be used to evaluate the degree of dynamic geometrical nonlinearity quantitatively for the postbuckled plate with symmetric snap-through motion.

1. Introduction

In order to protect substructure of hypersonic flight vehicle, a two-layer structure with a thin high temperature-resistant ceramic-based composite surface plate adhesively bonded to a thick aerogel thermal insulator has been developed as a thermal protection system (TPS) (Liu et al., 2014; Zhang et al., 2015; Li and Yu, 2015; Liu and Li, 2013; Du et al., 2016; Zhao et al., 2017). Generally, the top facesheet of TPS is subjected to high-intensity acoustic loading from sources such as jet efflux and turbulent fluid flow. Frequently, this noise is combined with elevated temperature environment. Some thin surface panels excited by the intense acoustic and thermal loadings (~1650 °C temperature and ~180 dB Overall Sound Pressure Level, OASPL) can exhibit complex structural response characteristic, such as an erratic fully nonlinear snap-through behavior (Chung Fai, 1989; Chung Fai and Clevenson, 2012; Murphy et al., 1996). The nonlinearity alters the dynamic displacement, strain and stress responses, which in turn affect the high-cycle fatigue life (Vaicaitis, 1994; Sha et al., 2012; Dhainaut and Mei, 2006; Przekop et al., 2003; Locke and Mei, 1990). Considerable efforts have been made to develop models and approaches to study nonlinear dynamic response of panel-type structures subjected to a combination of thermal and acoustic loadings.

Extensive researches have been performed by reduced-order method (ROM) to investigate nonlinear dynamic response for thin panel-type structures subjected to complex loadings. The works of Nash (1977), Shi and Mei (1996), McEwan et al. (2001a, 2001b), Hollkamp et al. (2003, 2005), Mignolet et al. (2003), Przekop and Rizzi (2006), and Muravyov and Rizzi (2003) are good examples of generating reduced models based solely on finite element (FE) analysis results. In these cases, nonlinear reduced models are assembled using numerically identified nonlinear coefficients through direct or indirect nonlinear modal stiffness evaluation methods. For instance, nonlinear reduced models were obtained through manipulation of the FE stiffness matrices in Nash (1977) and Shi and Mei (1996). The focuses of the remaining work were to identify constant, nonlinear stiffness coefficients indirectly from a series of nonlinear FE static analyses using generic FE algorithms and to select optimized modal basis in nonlinear reduced-

* Corresponding author.
E-mail address: liuliu@bit.edu.cn (L. Liu).

https://doi.org/10.1016/j.ijsolstr.2018.03.015
0020-7683/© 2018 Elsevier Ltd. All rights reserved.

Nomenclature

L_x	plate length
L_y	plate width
h	plate thickness
x	coordinate axis parallel to the plate length
y	coordinate axis parallel to the plate width
z	coordinate axis parallel to the plate thickness
$\bar{K}_1$	Winkler foundation stiffness
$\bar{K}_2$	shear layer stiffness of the foundation
$p_0(t)$	a uniform distribution of random acoustic pressure
$\{u, v, w\}^t$	displacements along x, y and z direction
$\{\bar{u}, \bar{v}, \bar{w}\}^t$	midplane displacements along the x, y and z directions
$\{\overline{\varepsilon}_x, \overline{\varepsilon}_y, \overline{\gamma}_{xy}\}^t$	midplane in-plane strain
$\{\kappa_x, \kappa_y, \kappa_{xy}\}^t$	curvature of the plate
$\{\alpha_x, \alpha_y, \alpha_{xy}\}^T$	thermal expansion coefficients along the x, y and xy directions
$[\bar{\boldsymbol{Q}}]$	transformed reduced stiffness matrix
$\bar{T}(x, y)$	averaged mid-plane temperature distribution function
$\{N_x, N_y, N_{xy}\}^t$	resultant in-plane membrane force vector
$\{M_x, M_y, M_{xy}\}^t$	resultant bending moment vector
$[\boldsymbol{A}]$, $[\boldsymbol{B}]$, $[\boldsymbol{D}]$	structural extensional, coupling and bending stiffness matrices
$\{N_x^T, N_y^T, N_{xy}^T\}^t$	resultant in-plane thermal force vector
$F(x, y)$	Airy stress function
ρ	density of material
c	viscous damping coefficient
$q_r(t)$	plate modal amplitude
$\phi_r(x,y)$	plate mode function
R	index of plate mode
t_0	uniform temperature rise above reference temperature
δ_v	scaling factor to uniform temperature rise t_0
f_{pq}	coefficient in particular part of Airy stress function
$\boldsymbol{C}$	linear viscous damping matrix
$\boldsymbol{K}$	linear structural stiffness matrix
$\boldsymbol{G}$	linear stiffness matrix induced by uniform temperature rising
$\boldsymbol{H}$	linear stiffness matrix induced by non-uniform temperature variation
$\boldsymbol{a}$	nonlinear cubic modal amplitude term
ξ_{mn}	modal damping ratio in the mnth mode measured experimentally
ω_{mn}	resonant frequency of the mnth mode
β	L_y/L_x, the aspect ratio of the plate
b_0	constant term in the in-plane strain
s_i	coefficient of the linear term in the in-plane strain
d_{ij}	coefficient of the quadratic term in the in-plane strain
T_{cr}	critical buckling temperature
λ	skewness of the probability distribution
γ	kurtosis of the probability distribution
M_2	second central moment of the probability distribution
M_3	third central moment of the probability distribution
M_4	forth cumulant of the probability distribution

order simulations (Rizzi and Przekop, 2013; Przekop et al., 2012). Spottswood and Allemang (2006) developed a nonlinear parameters identification method in the reduced-order space using experimental data. Reduced-order models provide predictions of dynamic displacement response, which is the primary variable. Stress and strain are derived through "stress recovery" or others methods (Przekop and Rizzi, 2006; Mei et al., 2000).

There are considerable researches related to nonlinear dynamic response analyses using equivalent linearization method, such as random fatigue prediction of thin-walled structures subjected to normally distributed loadings (Sun and Miles, 1991; Sun et al., 1998, 2001). Further, the effects of non-Gaussian loadings on the nonlinear dynamic response of thin-walled structures are investigated (Dhainaut and Mei, 2006; Przekop et al., 2003; Locke and Mei, 1990; Sweitzer, 2006; Steinwolf et al., 2000; Rizzi et al., 2010). Recorded aircraft acoustic pressure data with nonwhite power spectral density can yield higher stress characteristics and shorter fatigue life than corresponding equivalent white noise data (Dhainaut and Mei, 2006; Przekop et al., 2003; Locke and Mei, 1990). High-cycle fatigue of a beam structure under a combination of thermal and non-Gaussian acoustic loadings has been considered using ABAQUS/Explicit solution by Rizzi et al. (2010). All results show that non-Gaussian loading could accelerate fatigue damage accumulation and reduce fatigue life compared to Gaussian loading case.

In order to predict displacement and strain statistics for thermally buckled plates, Lee (1993, 1997, 2001) and Lee et al. (1998) used Galerkin method and stationary Fokker–Planck distribution to initiate a study of a single-mode equation for a thermally buckled composite plate. From the single-mode Galerkin representation of the nonlinear plate equation, it has been demonstrated that not only the root-mean-square (RMS) displacement and strain are independent of the sound pressure at high temperature, but also they obey simple temperature relations as the plate temperature rises since the statistical panel dynamics are governed by the static postbuckled displacement at high temperature. The skewed strain histogram with two unequal peaks has been exhibited (Lee, 1993). However prediction based on the single-mode model is not accurate enough for the structure with multi-mode dynamic behavior in thermal environments (Muravyov and Rizzi, 2003) and mechanism for such a non-Gaussian strain histogram has not been clarified. The deflection and strain statistics for large amplitude random response of panels subjected to acoustic excitation and temperature have been studied using an efficient finite element time-domain modal formulation by Dhainaut and Mei (2006), Przekop et al. (2003), and Locke and Mei (1990). Their results indicate that PDF of the maximum stress deviates largely from a Gaussian distribution, and the peak PDF for nonlinear maximum stress is non-Rayleigh.

Very few analyses and results have been reported in open literatures regarding characterizations of strain process for thin panel-type structures subject to a combination of thermal and acoustic loadings. Strain time history is available in most structural dynamic experiments and related to fatigue life prediction (Dhainaut and Mei, 2006; Przekop et al., 2003; Locke and Mei, 1990; Sweitzer, 2006; Steinwolf et al., 2000; Rizzi et al., 2010). Therefore it is of interest to investigate displacement and strain statistics of nonlinear dynamic response for thin panel-type structure under thermal and acoustic loadings. Characteristics of displacement and strain process in time and frequency domain for pre- and postbuckled plate are desirous. Indication of snap-through motion and assessment of degree of geometrical nonlinearity through experimental data, such as strain time history, are valuable. However they have not been addressed too much.

The effort of this work focuses on the dynamic response of the thin facesheet of a two-layer TPS subjected to a combination of