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Strain gradient fracture in an anti-plane cracked material layer

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ABSTRACT

Fracture mechanics analysis in terms of evaluating stress intensity factors of an anti-plane isotropic cracked layer is carried out using strain gradient elasticity theory. The crack plane is assumed parallel to the layer edges. Both volumetric and surface strain gradient material characteristic lengths are considered in formulations and numerical solutions. Two boundary value problems corresponding to “stress-free” and “clamped” boundaries are considered in which each solution is reduced to the dual integral equations. The Fredholm integral equation, proceeding from the dual integral equations, is numerically solved to evaluate crack tip stress intensity factor. Stress intensity factors for stress-free boundary conditions are higher with smaller height (or with a longer crack) and vice versa for clamped boundaries. Volumetric strain gradient effect reduces stress intensity factor and demonstrates strong size effect on a smaller scale. Crack stiffness becomes more pronounced with positive surface strain gradient, while negative surface gradient leads to a more complaint crack. In general, the contribution of volumetric strain gradient is shown to be more dominant than that of surface strain gradient.

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1. Introduction

Material discontinuities in micro and nanoscale structures promote its mechanical behavior to be size dependent. This size dependency (size effect) has been thoroughly observed in experimental studies (Fleck & Hutchinson, 1997; Lam et al., 2003; McFarland & Colton, 2005). The discrete nature of the material medium (at a smaller scale) is not considered in classical continuum theories where internal dimensions of the structure are assumed negligible in comparison to the external ones. Hence, material's elastic as well as the plastic behavior becomes scale-free and independent of an underlying microstructure. Over the years, several theories have been proposed incorporating intrinsic length scale in the continuum model to cater for size effect. Some of the well-known theories in this regard are non-local elasticity theory (Eringen and Edelen, 1972), couple stress theory (Yang et al., 2002) and strain gradient theory (Aifantis, 1992, 2003; Lam et al., 2003). Pioneer work related to strain gradient theory was first postulated by Mindlin (1964, 1965) and further re-established and updated by Aifantis in (1992) and (2003), respectively. The physical meaning of higher order strain tensor employed in gradient elasticity theories is recently provided by Polizzotto (2016). On the other hand, the strain gradient theory proposed by Vardoulakis et al. (1996), provides the simplest and most practical generalization of corresponding constitutive theory accounting for only two material characteristic lengths (with the units of length). These material constants are responsible for material volumetric and surface strain gradient terms, usually represented as l and l', respectively (Chan et al., 2008; Paulino et al., 2003; Vardoulakis et al., 1996). This theory has been successfully employed to observe size effects (Aifantis, 2011) in various engineering problems such as in twisted micro-wires and bent micro-cantilever beams (Aifantis, 1999). A comprehensive review of this gradient theory and applications of an internal length gradient across various scales is recently provided by Aifantis (2016). Further application and validation of this simpler strain gradient theory are confirmed by Vardoulakis and Sulem (1995) and Giannakopoulos and Stamoulis (2007). Very recently application can be found in the fracture study of double cantilever beam fracture mechanics specimen conducted by the authors (Joseph et al., 2017).

Quite a few studies related to the fracture problem in infinite medium (in which the geometric disturbance is only due to crack), based on gradient elasticity theories, are conducted over the years. For instance, one of the pioneering works in the field of gradient elasticity in Mode-III crack problem was conducted by Vardoulakis et al. (1996), Exadaktylos (1998) and subsequently by Exadaktylos and Vardoulakis (2001). In these papers, two material parameters l and l' related to volumetric and surface strain gradients were used to solve two boundary value problems i.e. traction boundary value problem and mixed boundary value problem. Paulino et al. (2003) and Chan et al. (2008) em-

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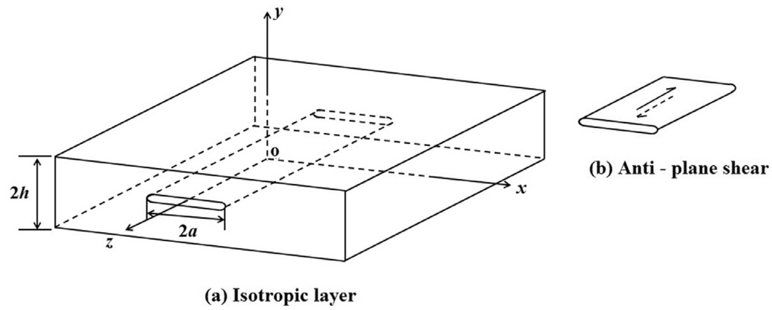

Fig. 1. Schematic diagrams.

ployed gradient elasticity theory to solve mode III crack problems in functionally graded materials. In the first case, crack is assumed perpendicular (Paulino et al., 2003), while in second case crack plane is parallel to the material gradation (Chan et al., 2008). Fannjiang et al. (2002) employed a hyper-singular integrodifferential equation approach to solve the anti-plane shear crack problem using strain gradient elasticity theory. Some interesting information related to dislocation based-gradient elastic fracture mechanics for the anti-plane crack problem is discussed by Mousavi and Aifantis (2015). A very comprehensive study related to an anti-plane analysis of an infinite plane with multiple cracks based on strain gradient theory is recently conducted by Karimipour and Fotuhi (2017).

Above studies are strictly related to the bodies whose edges are far away and any disturbance in the material medium is primarily due to crack initiation and propagation. However, in many practical engineering applications, the specimen boundaries are finite and hence contribute significantly to its fracture behavior. One such case is the crack initiation and propagation in elastic media of the form of layer/strip with finite height. Estimation of fracture properties in this case, such as stress intensity factor at the crack tip, is extremely vital to accurately predict crack growth rates. For classical case, several analytical models have been proposed by researchers using different approaches. For instance, closed form solutions of a crack at the mid-plane of elastic media subjected to anti-plane shear stress are obtained by (Yang, 1997). Singh et al. (1981) employed Fourier transform technique, while Tait and Moodie (1981) utilized the complex variable method to provide the closed form solution of mode-III crack moving along the center of an elastic strip. On the other hand, the solution of interface crack between two dissimilar materials in a closed form is provided by Li (2001). For an anti-plane interface crack between two dissimilar magneto-electroelastic layers, Wang and Mai (2006) evaluated closed-form solutions for stress intensity factors. Furthermore, thorough investigations of mode-III crack in multi-layered composites are provided by Sih and Chen (1981).

According to authors' knowledge, most of the models in literature may be divided into two groups; The first group relates to the evaluation of closed form solution of an anti-plane crack in an infinite medium based on gradient elasticity theories and the second group comprises those classical studies which are conducted to evaluate the closed form solution of cracked elastic strip/layer (finite boundaries). Therefore, in this article the simplest strain gradient theory (proposed by Vardoulakis et al. 1996, Exadaktylos, 1998 and subsequently elaborated by Exadaktylos and Vardoulakis, 2001) comprising two material parameters, related to volumetric and surface strain gradients, respectively, is applied to solve the mode III crack problem in an elastic isotropic layer. The objective here is to numerically estimate the crack tip stress intensity factors of a crack propagated at the middle plane of an elastic isotropic layer having finite height. Two types of boundary value problems are considered i.e. stress-free boundaries and clamped boundaries. The solution of each problem is reduced to dual integral equations. The kernel of the Fredholm integral equation (an improper integral that ranges from 0 to infinity) of the second kind, thus obtained (by applying the method of Copson (1961) on the dual integral equations) is numerically solved by using the collocation method of Gauss–Laguerre quadrature.

2. Theoretical formulations

This section introduces the constitutive equations and theoretical formulations of an isotropic elastic layer of finite thickness using strain gradient theory. Firstly, the constitutive equations are derived without incorporating surface strain gradient effect ($l'=0$) followed by the detailed analysis of complete strain gradient model (with both volumetric and surface strain gradient effect). The approach considered in this paper is similar to one adopted by Vardoulakis et al. (1996) and Exadaktylos (1998).

Consider a crack of length $2a$ placed at the mid plane of an isotropic layer with thickness (height) $2h$. The boundaries of the layer are at $y=\pm h$ and reference axes are shown in Fig. 1a. The crack surfaces are subjected to the applied anti-plane shear stress (τ_1) as shown in Fig. 1b. The conditions at $y=0$ are given as: $\tau_{yz}(x,0)=-\tau_1$ for $|x|<a$ and $w_z(x,0)=0$ for $|x|\geq a$. For the upper half plane i.e. $y\geq 0$, the stresses and double stress derived from the constitutive equations of gradient elasticity with surface energy are given as (Chan et al., 2008; Vardoulakis et al., 1996)

$$\tau_{zx}=G\left[\frac{\partial w_z}{\partial x}-l^2\nabla^2\frac{\partial w_z}{\partial x}\right] \tag{1a}$$

$$\tau_{yz}=G\left[\frac{\partial w_z}{\partial y}-l^2\nabla^2\frac{\partial w_z}{\partial y}\right] \tag{1b}$$

$$\mu_{xxz}=Gl^2\frac{\partial^2 w_z}{\partial x^2} \tag{1c}$$

$$\mu_{xyz}=Gl^2\frac{\partial^2 w_z}{\partial x\partial y} \tag{1d}$$

$$\mu_{yxz}=G\left[-l'\frac{\partial w_z}{\partial x}+l^2\frac{\partial^2 w_z}{\partial x\partial y}\right] \tag{1e}$$

$$\mu_{yyz}=G\left[-l'\frac{\partial w_z}{\partial y}+l^2\frac{\partial^2 w_z}{\partial y^2}\right] \tag{1f}$$

Here $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$, l and l' are the volumetric and surface material characteristic lengths, respectively. For an anti-plane shear crack problem as depicted in Fig. 1, we have $u_x=u_y=0$, $u_z\neq 0$. Also $\sigma_{xx}=\sigma_{yy}=\sigma_{zz}=\tau_{xy}=0$ while $\tau_{yz}\neq 0$ and $\tau_{xz}\neq 0$. The equilibrium equation i.e. $\partial\tau_{yz}/\partial y+\partial\tau_{xz}/\partial x=0$ with the help of equations in (1) may be expanded as

$$\nabla^2 w_z-l^2\nabla^4 w_z=0 \tag{2a}$$

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