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On micromechanical modeling of orthotropic solids with parallel cracks

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ABSTRACT

A detailed comparison of the accuracy of several popular micromechanical schemes utilized for prediction of the effective elastic properties of materials with parallel cracks is presented. In particular, the non-interaction, Mori–Tanaka, differential and self-consistent schemes are compared against the direct finite element simulations. The latter are performed on the periodic representative volume elements containing 30 strongly oblate spheroids representing the penny-shaped cracks. This work extends the existent results to a more general class of matrix materials – orthotropic materials, which requires the ability to calculate the Eshelby tensor for an ellipsoid in non-isotropic matrix. In addition to the implementation of the integration procedure used for the Eshelby tensor calculation, this work also presents a variation of the Random Sequential Adsorption algorithm modified for periodic structures.

Analysis of the results indicates that in the case of parallel nearly flat cracks (strongly oblate spheroids) the overall out-of-plane moduli are best predicted by the differential scheme. On the other hand, Mori–Tanaka scheme should be used for estimation of the in-plane moduli. It also appears that as the cracks are inflated from strongly oblate spheroids to slightly deformed spheres, the best choice of the micromechanical scheme for the out-of-plane properties gradually shifts towards Mori–Tanaka.

1. Introduction

Micromechanical schemes, based on certain simplifying assumptions with respect to interaction of inhomogeneities, provide an efficient tool to predict the overall (effective) elastic properties of materials with inhomogeneities (Mori and Tanaka, 1973; Benveniste, 1987; Kröner, 1958; Hill, 1965; Budiansky, 1965; McLaughlin, 1977; Salganik, 1973; Eroshkin and Tsukrov, 2005). They are particularly suited for parametric studies focusing on evaluation of the effects of inhomogeneities shapes, concentration, and stiffness or compliance on the overall response. For such parametric studies, direct numerical simulations of a large number of the representative volume elements (RVE) of the materials with inhomogeneities would require significant computational resources and are not always feasible. Note that the third popular micromechanical approach involving establishing exact variational bounds on the overall elastic properties (Hashin and Shtrikman, 1963) may result in the bounds that are too wide in the case of large contrast in the elastic properties of matrix and inhomogeneities, e.g. pores and cracks.

In this paper we investigate the accuracy of several popular micromechanical schemes for isotropic and orthotropic solids with parallel penny-shaped cracks. This is done by comparing the analytical micromechanical modeling results with direct finite element (FE) simulations conducted on the RVEs of the cracked solids. We also consider how the accuracy of the schemes changes if the cracks in the solids are “inflated” to become the oblate spheroidal pores.

Evaluation of contribution of cracks to the effective elastic properties of cracked solids is one of the classical and well developed topics in the mechanics of solids, as reviewed, for example, in Kachanov (1993). However, certain controversy exists in the choice of the best micromechanical modeling approach to this problem, in particular, Mori–Tanaka (MT) (Mori and Tanaka, 1973; Benveniste, 1987) or differential (DIFF) scheme (Salganik, 1973; McLaughlin, 1977; Zimmerman, 1985). Note that in the case of cracks the predictions of Mori–Tanaka approach coincide with the non-interaction (NI) approximation, see Kachanov (1993). A number publications argue in favor of NI (and hence MT) scheme, see Kachanov (1993), Kachanov et al., (1994) and Grechka and Kachanov (2006). Their conclusions are based on comparison with

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numerical results utilizing the "transmission factors" approach of Kachanov (1987) or finite element simulations. On the other hand, in studies of several other research groups (Dahm and Becker, 1998; Orlowsky et al., 2003; Saenger et al., 2006) the differential scheme was found to be closer to numerical results. Saenger (2007) claimed that the results of Grechka (2007) favor the non-interaction approximation over differential scheme due to unjustified constraints on the location of cracks within RVE. Seelig et al. (2000) implemented the "transmission factor" approach of Kachanov (1987) in combination with the boundary element method. Their numerical results for random and parallel 2D cracks are located between the NI and DIFF predictions.

All of the above publications deal with cracks in isotropic matrices. For anisotropic solids with cracks, Gottesman et al. (1980) considered influence of parallel cracks on a fibrous composite material on the basis of variational techniques and also in the self-consistent scheme framework. Laws et al. (1983) presented results for transversely isotropic material with parallel cracks infinitely long in the direction of transverse isotropy axis (plane strain formulation). The cracks were modeled as elliptical cylinders; the results were obtained using self-consistent method. Hashin (1988) considered isotropic material with randomly oriented cracks and 2D case of orthotropic material with parallel cracks utilizing differential and self-consistent methods. Deng and Nemat-Nasser (1992) considered anisotropic material with parallel cracks in 2D using dilute distribution, differential and self-consistent methods. Mauge and Kachanov (1994) performed numerical simulations on sample arrays of parallel and randomly oriented interacting cracks in 2D materials with random anisotropy and compared the calculated values of Young's moduli with predictions given by non-interaction, self-consistent and differential schemes. Their simulations were limited by the requirement that neighboring cracks were not closer than 0.2 of crack lengths to have convergence in the utilized numerical procedure.

3D results for anisotropic matrix are mostly limited to transversely isotropic materials with cracks parallel to the plane of isotropy. Willis (1977) presented expressions for Hashin–Shtrikman bounds and self-consistent approach for composite with anisotropic constituents and illustrated the results by calculating the effective conductivity of an anisotropic matrix with aligned spheroidal inhomogeneities. Withers (1989) derived a solution for elastic fields around an ellipsoidal inclusion in transversely isotropic matrix using a method analogous to Eshelby (1957). Kushch and Sevostianov (2004) presented results for effective elastic properties of composites with spherical inhomogeneities and transversely isotropic material properties obtained using an analytical approach based on the multipole expansion method. Sevostianov et al. (2005) performed a micromechanical modeling based on the stiffness and compliance contribution tensors combined with non-interaction and effective field method for composites with spheroidal inhomogeneities and transversely isotropic phases.

In this paper we investigate the applicability of several commonly used micromechanical schemes to prediction of effective elastic response of orthotropic materials containing parallel penny-shaped crack-like pores. The paper is organized as follows. Section 2 presents our approach to micromechanical modeling based on contribution of a single inhomogeneity to the effective elastic properties combined with non-interaction, Mori–Tanaka, differential and self-consistent schemes. The procedures for geometry generation, meshing, FE model preparation and processing of simulation results are described in section 3. This section includes a mesh sensitivity study, in which we compare the effect of two mesh refinement techniques (local and global) and the order of the volumetric finite elements on the elastic moduli predictions. Section 4 focuses on the performance of the micromechanical schemes for orthotropic materials containing parallel crack-like pores. The section also contains a validation of the numerical procedure for calculating the Eshelby tensor for orthotropic materials which is required for the proposed micromechanical modeling. Section 5 presents a study on the accuracy of the considered micromechanical schemes for isotropic materials with oblate spheroidal pores having aspect ratios in the range 0.1–0.8. Final conclusions of this research are formulated in Section 6.

2. Micromechanical modeling

The micromechanical approach utilized in this paper is based on the compliance contribution tensor of inhomogeneities $\mathbf{H}$. This concept was first introduced in Horii and Nemat-Nasser (1983) and then used in Kachanov et al. (1994) to evaluate contributions of various pores in isotropic 2D and 3D materials. The approach is presented here for materials with inhomogeneities of general type and shape following Eroshkin and Tsukrov (2005). In the case of pores and cracks the stiffness of the inhomogeneity is equal to zero.

In this work we assume that the cracked material is statistically homogenous and a certain representative volume element (RVE) can be chosen such that its properties are the same as of the entire heterogeneous material. More detail on the concept of the RVE can be found in Hill (1963) and Markov (1999).

The effective compliance $\mathbf{S}$ of a material with inhomogeneities is expressed as

$$\mathbf{S} = \mathbf{S}_{\mathrm{M}} + \mathbf{H}^{\mathrm{RVE}}, \tag{1}$$

where $\mathbf{S}_{\mathrm{M}}$ is the compliance tensor of the matrix material and $\mathbf{H}^{\mathrm{RVE}}$ is the compliance contribution tensor of all inhomogeneities present in the RVE. For non-interacting defects tensor $\mathbf{H}^{\mathrm{RVE}}$ can be found simply as a sum of contributions of individual defects. The approaches to deal with interacting defects are discussed later in this section.

In order to obtain contributions of various types of defects, we need to solve the so-called "single inclusion problem". Contribution of an inhomogeneity to the overall properties of a material is evaluated as follows. The inhomogeneity of volume V_{I} is placed in an infinite elastic matrix subjected to the remotely applied stress $\boldsymbol{\sigma}^{\infty}$. The additional average strain in some reference volume $\tilde{V}$ containing inhomogeneity is proportional to the applied stress:

$$\Delta\boldsymbol{\varepsilon} = \mathbf{H} : \boldsymbol{\sigma}^{\infty}$$

where $\mathbf{H}$ is the inhomogeneity compliance contribution tensor.

Most of the micromechanical models are based on the solution for the ellipsoidal inhomogeneity provided by Eshelby (1957). Originally the solution was derived for the isotropic material in terms of elliptic integrals for a general ellipsoidal inhomogeneity. Later this solution was expanded to other types of matrix material symmetry, see, for example, Sevostianov et al. (2005). However, there is still no explicit analytical solution of the Eshelby problem for a general anisotropic material. This issue was addressed by Mura (1987), who proposed to simplify the elliptic integrals to triple integrals. The idea was implemented numerically by Ghahremani (1977) and improved by Gavazzi and Lagoudas (1990).

The expression for the compliance contribution tensor of an ellipsoidal inclusion $\mathbf{H}$ in terms of the compliance tensors of the matrix $\mathbf{S}_{\mathrm{M}}$ and the inhomogeneity $\mathbf{S}_{\mathrm{I}}$ and Eshelby tensor $\mathbf{s}$ can be written as (see Sevostianov and Kachanov, 2002)

$$\mathbf{H} = \frac{V_{\mathrm{I}}}{\tilde{V}}\left[(\mathbf{S}_{\mathrm{I}} - \mathbf{S}_{\mathrm{M}})^{-1} + \mathbf{S}_{\mathrm{M}}^{-1} : (\mathbf{I} - \mathbf{s})\right]. \tag{2}$$

It is extremely difficult to analytically solve the elasticity problem for multiple interacting inhomogeneities. So, the interaction between defects is taken into account utilizing various simplifying micromechanical models (or schemes). Depending on how