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Effective elastic properties of a periodic fiber reinforced composite with parallelogram-like arrangement of fibers and imperfect contact between matrix and fibers

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ABSTRACT

The paper focuses on application of asymptotic homogenization method (AHM) to calculation of the effective elastic constants for fiber reinforced periodic composite with imperfect contact conditions between fibers and matrix. The arrangement of the fibers is assumed to be parallelogram like and imperfectness of the contact is modeled by linear springs. This work is an extension of previously reported results of López-Realpozo et al. (2011), where perfect contact between the phases of the composite with parallelogram cells has been considered. The constituents of the composite are assumed to possess coaxial transversely isotropic properties. The obtained results are compared with some numerical examples of Hui-Zu and Tsu-Wei (1995), with the differential approach of Sevostianov and Kachanov (2007) and with experimental results.

1. Introduction

Most of the presently developed micromechanical models of fiber-reinforced composites operate with perfect fiber–matrix bond and assume continuity of displacements and normal stress across the interface. However, experiments show that local or partial debonding at interfaces is a rule rather than the exception in composites (Rokhlin et al., 1994; Hui-Zu and Tsu-Wei, 1995). Another imperfectness is related to formation of the third phase during manufacturing process due to chemical treatments of fiber surfaces and partial resin crystallization (Achenbach and Zhu, 1989). To describe these imperfectnesses, several approaches have been developed in which the bond between the inhomogeneities and the matrix is modeled by an interphase zone of certain thickness and elastic properties.

Mathematical analyses of inhomogeneous interfaces started, probably, with the work of Kanaun and Kudriavtseva (1983, 1986) on the effective elasticity of a medium with spherical and cylindrical inclusions, correspondingly, surrounded by radially inhomogeneous interphase zones. In these papers, the basic idea of replacing an inhomogeneous inclusion by an equivalent homogeneous one has been formulated. Such a replacement was carried out by modeling the inhomogeneous interface by a number of thin concentric layers. The effective elastic properties of a composite containing a finite concentration of inclusions was found by replacing inhomogeneous inclusions by equivalent homogeneous ones (with properties found by the multilayer approximation) and applying the effective field method, whereby each inclusion is placed in a certain effective stress.

The ideas developed by Kanaun and Kudriavtseva (1983, 1986) have appeared in a number of later works (see, for example, Herve and Zaoui, 1993). The basic idea of replacing inhomogeneous inclusions by equivalent homogeneous ones has been utilized in the majority of works on the topic. Theocaris (1985) and Theocaris and Varias (1986) considered the mesophase layer as an independent phase of variable properties, matching those of the inclusion on one side and the matrix on the other. The examples of laws of variation of the Young's modulus E and Poisson's ratio ν across the interphase layer include linear, parabolic, hyperbolic and a logarithmic ones. Theocaris and Varias (1986) described a model predicting the influence of the mesophase on effective properties of fibrous composites. Their model is based on a corrected version of Kerner's model (see Christensen, 1979). Pagano and Tandon (1988, 1990) developed two models to approximate the

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thermoelastic response of a composite body reinforced by coated fibers oriented in various directions. The fundamental representative volume element used in these papers is a three-phase concentric circular cylinder subjected to prescribed displacements and surface tractions. The analysis leads to estimation of the effect of interphase layers on the effective thermoelastic properties of fiber reinforced composites.

Sutcu (1992) presented a simple recursive algorithm which accounts for two concentric cylinders at a time, in order to calculate five effective elastic constants and two linear thermal expansion coefficients for a uniaxially aligned composite that contains an arbitrary number of coatings on its fibers. The effective elastic properties are calculated using expressions for two phase composites proposed by Hashin (1979) and Christensen (1979). Dasgupta and Bhandarkar (1992) discussed a method to obtain the transversely isotropic effective thermomechanical properties of unidirectional composites reinforced with coated cylindrical fibers. In this work the method developed by Benveniste et al. (1989) is applied to composites with multiply-coated cylindrical inhomogeneities. Lagache et al. (1994) determined numerically the effect of a mesophase using a finite element formulation in order to solve the local problems derived from the homogenization method. Chu and Rokhlin (1995) suggested a method for the inverse determination of effective elastic moduli of mesophases using a multiphase generalized self-consistent model.

The idea of approximating radially variable properties by multiple layers (piecewise constant variation of properties) was explored by Garboczi and Bentz (1997) and Garboczi and Berryman (2000) in the context of applications to concrete composites. An alternative method was used by Jasiuk and Tong (1989), Jasiuk et al. (1992) and Wang and Jasiuk (1998). They considered a general composite material with spherical inclusions representing the interphase as a functionally graded material and calculated effective elastic moduli using the composite spheres assemblage method for the effective bulk modulus and the generalized self-consistent method for the effective shear modulus.

Several closed form solutions for two specific forms of radial variation of properties – the linear and the power law ones – have been produced. Lutz and Ferrari (1993) and Zimmerman and Lutz (1999) considered inclusions with linearly varying elastic moduli, in the context of the effective bulk modulus. Lutz and Zimmerman (1996a) considered the linear variation of the thermal expansion coefficient. Lutz and Zimmerman (1996b, 2005) and Lutz et al. (1997) considered the power law variation, in the context of effective bulk modulus and effective conductivity.

Alternative approach has been proposed by Hashin (1991a,b, 2002). He suggested to consider the interphase as a boundary layer of zero thickness with discontinuity of the displacements across this layer. More recently, Caporale et al. (2006) used finite elements analysis to study mechanical behavior of unidirectional fiber reinforced composites with imperfect interfacial bonding. In this work, an interfacial failure model is implemented by connecting the fibers and the matrix at the finite element nodes by normal and tangential brittle-elastic springs. Bisegna and Caselli (2008) and Artioli et al. (2010) obtained closed-form expression for the homogenized longitudinal shear moduli of a linear elastic composite material reinforced by long, parallel, circular fibres with a periodic arrangement and imperfect (linear) fiber–matrix interface.

In the present paper we consider composite with parallelogram-like arrangement of fibers and discuss effect of the interface imperfectness on the overall elastic moduli. The analytical expressions are obtained by two scale asymptotic homogenization method (see books of Sanchez-Palencia, 1980; Pobedria, 1984; Bakhvalov and Panasenko, 1989). This work continues previous study of Guinovart-Diaz et al. (2011) where only perfect contact has been considered and Sevostianov et al. (2012), where square arrangement of fibers with special type of the interface imperfectness (incompressible layer between the phases) has been discussed. The novelty of the present analysis is that the formulation of the local problems for linear two phase elastic composites with spring imperfect contact conditions is given in general form. The solution for each plane local problems is found using the potential methods of a complex variable and the properties of doubly periodic Weierstrass elliptic functions. The complete set of effective elastic constants is obtained in explicit formusing the asymptotic homogenization method (AHM) for fiber reinforced composites with periodic parallelogram cell of circular cylindrical shape periodically distributed in the matrix under imperfect contact conditions. The results are compared with some numerical examples and with the results obtained by differential approach developed by Sevostianov (2007) and Sevostianov and Kachanov (2007) extended to parallelogram periodic cell. In this case, the isotropic interphase properties are related to the thickness, volume fraction and the spring imperfect parameters.

2. Statement of the problem. Local problems based on asymptotic homogenization method

Nowadays, a wide class of materials exhibit a general anisotropic behavior, for instance, piezoelectric materials among others, which can not be isotropic and they are mainly transversely isotropic. Therefore, the study of composites with more general anisotropic character in the constituents, such as, transversely isotropic is important as an intermediate study between elastic composites with isotropic phases and composites with piezoelectric phases. Then, in the present work we consider a two phase linear elastic materials with both phases being transversely-isotropic; the axis of transverse symmetry coincides with the fiber direction, which is taken as the Ox_3 axis. The fibers of circular cross-section are periodically distributed without overlapping in directions parallel to the Ow_1- and Ow_2-axis, where $w_1 \neq 0$ and $w_2 \neq 0 (w_2 \neq \lambda w_1,\ \lambda \in \mathbb{R})$ are two complex numbers which define the parallelogram periodic cell of the two-phase composite. As shown in Fig. 1, the infinitely extended doubly-periodic structure is obtained from an elementary cell which is repeated in the two directions with fundamental periods w_1 and w_2. The general period P_{st} can be defined as $P_{st} = sw_1 +$ -

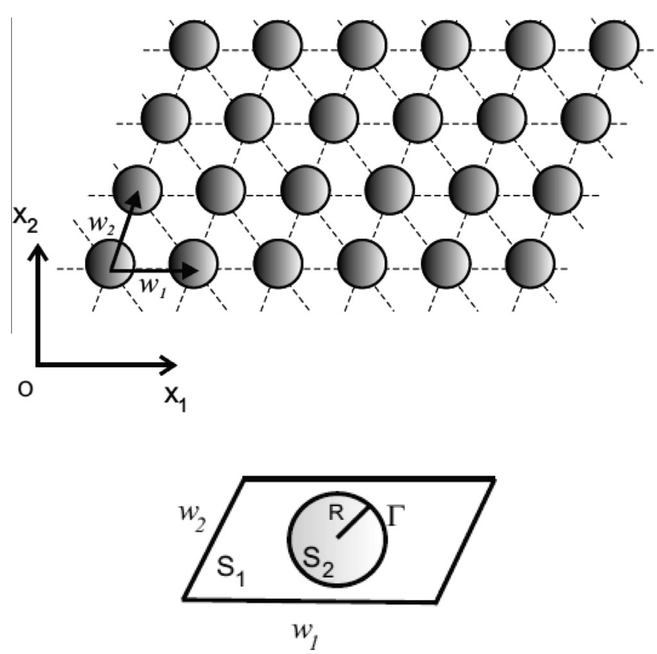

Fig. 1. The cross section of a quadratic and periodic array of circular fibers.