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Design of the optimal fiber-reinforcement for masonry structures via topology optimization

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ABSTRACT

A novel approach for the rational positioning of fiber reinforcements on masonry structures based on topology optimization is presented. Due to the brittle behavior of masonry, the minimization of the strain energy cannot be implemented to generate truss-like layouts that may be interpreted as strut-and-tie models in the discontinuity regions of reinforced concrete structures. To cope with the brittleness of brickwork, the optimal problem can be conveniently reduced to the minimization of the amount of reinforcement required to keep tensile stresses in any masonry element below a prescribed threshold. A strength criterion recently proposed for masonry is employed, based on a lower bound limit analysis homogenization model (Milani, 2011) and relying upon a discretization of ¼ of any unit cell by six CST elements. Thanks to the limited number of variables involved, closed form solutions for the masonry macroscopic strength domain can be obtained. This criterion is implemented into the multi-constrained discrete formulation of the topology optimization algorithm, to locally control the stress field over the design domain. For comparison, the phenomenological Tsai–Wu strength criterion for anisotropic solids is also implemented.

The contribution discusses three sets of numerical results, addressing the fiber-reinforcement of some benchmark masonry walls. The optimal reinforcement layouts are found to be affected by the choice of the masonry strength criterion only to a limited extent, as far as failure in the masonry element is mainly due to tensile stresses. Contrary to intuition, placing the reinforcing fibers along the direction of the principal tensile stresses in masonry is also found to be not necessarily the most effective solution, for certain geometries and load conditions.

1. Introduction

Making the built heritage safe is nowadays a problem of paramount importance, especially for countries, such as Italy, where the number of historic and monumental buildings of great artistic value is particularly important. These buildings are basically made of brick or stone masonry, which is well known to be a brittle material with negligible tensile strength: accordingly, they are extremely vulnerable to horizontal loads, ground settlements, etc. Preservation of the built heritage is important not only in terms of safety of the human life, but also for its fallouts in the field of civil construction, as well as in the tourism industry.

The use of Fiber Reinforced Polymers (FRPs) for the retrofitting of existing buildings has dramatically increased in the last decades, mostly because of the need of meeting current standards, or to protect any damaged structural element from further environmental aggression. Originally, this technique has been designed to retrofit concrete structures (Norris et al., 1997; Triantafillou, 1998): FRP strips or sheets are mostly employed to externally reinforce cracked r.c. beams (Arduini and Nanni, 1997), or to wrap columns to enhance their mechanical performances under horizontal actions (Shahawya et al., 2000). More recently, externally bonded FRP strips were also employed to retrofit or repair historic masonry buildings. This methodology has several advantages over standard retrofitting techniques, including flexibility, effectiveness and reversibility. Additionally, in the case of buildings in seismic regions, FRP strips do not significantly increase the structural mass and the earthquake-induced inertia forces.

Despite FRP strengthening can be conveniently employed for after shake rehabilitation or seismic upgrading of undamaged masonry structures, in presence of ground settlements this approach is potentially dangerous, since masonry structures might spontaneously find a new (safe) equilibrium configuration and the application of FRPs could lead them to ruinous collapse.

Laboratory tests and numerical models aimed at assessing the effectiveness of FRPs in enhancing the mechanical performances of masonry structures have been recently carried out on arches (Foraboschi, 2001), masonry walls monotonically loaded in various ways up to collapse (Grande et al., 2008; Milani, 2009, 2010) and

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walls subjected to cyclic loads (Capozucca, 2011). For a broad and quite updated overview of the experimental and numerical researches carried out on masonry structural elements reinforced by FRPs, readers are referred e.g. to Shrive (2006), Baratta and Corbi (2007), Elmalich and Rabinovitch (2009), Oliveira et al. (2010), Borri et al. (2011), Caporale and Luciano (2012), etc. A critical issue of the use of FRP strips in reinforcing masonry elements is the effectiveness of the interfacial bonding. Delamination between externally bonded FRP and masonry surfaces can nullify the strengthening effect of the reinforcement: this problem has been experimentally investigated e.g. by Aiello and Sciolti (2008) and Capozucca (2010). Appropriate surface treatments can avoid premature debonding at the masonry-FRP interface.

The analysis of the mechanical performances of masonry elements reinforced with FRP strips requires the utilization of suitably developed mathematical tools be used. Masonry is usually replaced by an anisotropic homogeneous equivalent continuum, whose mechanical properties can be derived by the theory of homogenization applied to periodic media in the case of brickwork with a regular pattern (see e.g. Anthoine, 1995; Pegon and Anthoine, 1997; Luciano and Sacco, 1998). Beyond the elastic limit, nonlinear effects such as plastic strains, strain softening, microcracking, and debonding must be taken into account. This was done e.g. by Luciano and Sacco (1998), who incorporated damage effects within a homogenization procedure for brickwork; an anisotropic Tsai–Hill-type strength criterion was employed for the FRP strips, where damage phenomena were also supposed to develop. Plastic strains, in addition to damage, in bricks and mortar are taken into account by Marfia and Sacco (2001), who homogenized masonry according to a 1D scheme; delamination and failure of the FRP strips are also taken into account. If only the load carrying capacity of the reinforced masonry elements is sought, limit analysis coupled with homogenization theory can be employed to derive the macroscopic strength properties of brickwork. This was done e.g. by Grande et al. (2008), who predicted the ultimate load of reinforced masonry walls tested in the laboratory. FRP strips were assumed to be brittle-elastic, and damage effects at the FRP-masonry interface were taken into account. An extension of this numerical procedure to multi-leaf walls was recently proposed by Milani (2010).

In the present work, attention is focused on in-plane loaded masonry walls. Out-of-plane failures, however, are quite common under horizontal actions, such as earthquakes, and are the main cause of collapse for buildings in seismic regions. This problem has been extensively dealt with by De Felice (2011), who carried out a thorough survey of the possible failure modes of unreinforced masonry walls under static and dynamic out-of-plane loads. Recently, Milani (2011) proposed failure surfaces for periodic brickwork under out-of-plane loads, which were employed to develop a theoretical model based on limit analysis to predict the collapse load of transversely loaded masonry walls strengthened by FRP strips.

So far, the layout of the reinforcing FRP strips placed on laboratory samples or real structures has been basically driven by the intuition, owing to the simplicity of the loading conditions, or by the intent of healing existing cracks. A more rigorous approach relying upon structural mechanics and optimization might be necessary under complex load conditions or geometries. A preliminary attempt toward a mechanically sound design of the reinforcing path was made by Krevaikas and Triantafillou (2005), who tried to identify on a rational basis the optimal layout of FRP strips on in-plane loaded masonry walls according to a strut-and-tie scheme.

Topology optimization has already been used to generate energy-based truss-like layouts that may be straightforwardly interpreted as strut-and-tie models in concrete structures. The minimization of the so-called structural compliance allows optimal load paths to be defined, which may inspire a safe disposal of the steel bars if the specimen is provided by the required ductility (see e.g. Liang et al., 2000; Bruggi, 2009). Due to the different behavior of masonry in tension and compression, the minimization of strain energy cannot be adopted as an objective. A stress-based approach allows the unequal properties of masonry in tension and compression to be dealt with through the implementation of ad-hoc strength criteria, with the aim of detecting regions where brittle behavior is to be expected. The problem may be therefore reformulated adopting the volume of the reinforcement (that is, the cost) as the objective function to be minimized, while enforcing constraints on the stress field in the masonry element. Affordable schemes for the stress-based optimization of continua adopt the amount of material as objective function, which is coupled with a set of local constraints over the domain. It can be shown that the energy-based and the stress-based approaches provide very similar results in presence of materials with symmetric behavior in tension and compression, see e.g. Bruggi and Duysinx (2012): accordingly, it seems reasonable to expect that the minimization of the amount of volume under stress constraints can be conveniently adopted as a tool to generate optimal load paths in structural elements made of materials with non-symmetric behavior.

This paper investigates the use of this formulation: the fiber-reinforcement of some benchmark masonry walls is addressed, with different assumptions on the strength criterion adopted to control the stress within the underlying brickwork.

The outline of the paper is as follows. First, some models proposed to describe the macroscopic strength properties of brickwork are recalled (Section 2). These include phenomenological-type strength criteria for orthotropic media (Section 2.1), and a model based on the theory of homogenization for periodic media, recently developed to predict the macroscopic strength of in-plane loaded masonry elements (Section 2.2). These models will be employed to check the admissibility of the stress at any point of the masonry element to be reinforced, resorting to a local stress-constrained problem (see e.g. Duysinx and Bendsøe, 1998). Then, the fundamentals of the adopted discrete formulation of topology optimization are briefly outlined; the mathematical problem to be solved to achieve the optimal layout of a unidirectional reinforcement to be bonded to any masonry element under given external loads is formulated (Section 3). This technique spontaneously leads to identify optimal reinforcement patterns that basically consist of ties. One of the advantages of topology optimization is that no a priori assumption regarding the position and the geometry of the reinforcing strips is required. The potentialities of the proposed approach are illustrated in Section 4 with reference to a few case studies. Finally, the main findings of the work are summarized and future perspectives of the research are outlined (Section 5).

2. Macroscopic strength domains for masonry

The topology optimization approach described in the following section will be used to predict the most suitable distribution of FRP that minimizes the volume of reinforcement to be applied to a given masonry structure, provided that the stress state at any point of the element fulfills certain conditions. This requires the adoption of a suitable model to describe the strength properties of the unreinforced brickwork.

In general, for a meaningful description of masonry at a structural level, a mechanical model should be capable of capturing the typical behavior exhibited by a brickwork panel in the elastic and inelastic range, namely orthotropy along the material symmetry axes and different strength properties in tension and compression.