International Journal of Solids and Structures 50 (2013) 1266–1284

Contents lists available at SciVerse ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

# Cohesive fracture of plane orthotropic layers

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ARTICLE INFO

*Article history:*
Received 23 April 2012
Received in revised form 27 November 2012
Available online 17 January 2013

*Keywords:*
Layers
Orthotropy
Debonding
Cohesion
Interface
Defect
Integral equation
Fracture
Elasticity

ABSTRACT

*Crack-like cohesive defect* propagation within a plane orthotropic linear elastic layer is considered by assuming that the defect, and its growth under load, can be modeled as the evolving separation along a straight, predetermined nonlinear, nonuniform Needleman-type cohesive interface. The analysis exploits a general form of orthotropy rescaling originally developed for the displacement boundary value problem by Krenk (1979). It is shown that when the material is *degenerate orthotropic* (i.e., $\rho = 1$, $\rho$ is the orthotropic shear parameter) rescaling enables the determination of solutions from isotropic ones and, when the material is fully orthotropic, rescaling allows for solutions to be obtained from problems with the simpler cubic symmetry. (These are well known attributes of linear *static sharp crack* analysis, which depend on an alternative form of rescaling the traction boundary value problem (Suo, 1990; Suo et al, 1991).) The procedure is demonstrated by obtaining degenerate orthotropic response from isotropic solutions recently obtained by the authors in an investigation of both solitary as well as multiple cohesive defect interaction problems in layered systems under arbitrary loading (Nguyen and Levy, 2009, 2011). In order to obtain fully orthotropic solutions via rescaling, a novel integral equation formulation is developed based on exact infinitesimal strain elasticity solutions for rectangular domains composed of cubically symmetric media and subject to arbitrary loading. Explicit results are obtained for the simple edge notch bend configuration, chosen so as to shed light on the mechanisms of defect propagation in orthotropic layers. It is demonstrated that increasing the orthotropic stiffness ratio can precipitate a quasi-brittle defect growth response. Furthermore, it is well known that in a number of technically important problem geometries and loadings, static sharp crack solutions are only weakly dependent on shear parameter $\rho$ enabling the estimation of fully orthotropic behavior from isotropic solutions (Suo et al, 1991). This result is shown to be true for nonlinear cohesive fracture analysis of the edge notch bend configuration analyzed in this study.



## 1. Introduction

The purpose of this paper is to extend an exact theory of nonlinear cohesive fracture of isotropic planar layers (Nguyen and Levy, 2009, 2011) to the realm of orthotropic elasticity. The technical significance of the work stems from the widespread use of composite layers, at least one of which is anisotropic, in adhesive and protective coatings (Chvedov and Jones, 2004; Graziano, 2000; Boelen et al. 2004), in dental restorations consisting of ceramic, ceramic filled polymer and cementitious layers (Niu et al, 2008) and in the rehabilitation of structures where fiber reinforced plastic plate is adhered to damaged concrete beams (Carpinteri et al, 2007; Wang, 2007; Au and Buyukozturk, 2006; Pan and Leung, 2007; Rabinovitch, 2008). Numerous other applications exist as well. The subject of this paper is cohesive fracture within a *single* orthotropic layer exclusively, while future work will address the heterogeneous multilayer cohesive interface fracture problem. The present analysis requires a straight nonlinear, nonuniform cohesive interface, along which a *crack-like defect*[1] will evolve, to be preselected to reside between two materially identical orthotropic sub-layers. Note that by nonlinear, nonuniform cohesive interface we mean an interface characterized by a traction-separation/slip relation that is a vector valued expression generally dependent on an interface coordinate dependent displacement jump vector and explicitly dependent on the interface coordinate through the interface strength. A well known example is the nonlinear exponential force law (Ferrante et al., 1982), which concerns normal separation only; given by $\mathbf{s}(\mathbf{n}; v) = e\sigma_{\max}\frac{v}{\delta}e^{-v/\delta}\mathbf{n}$ where $\mathbf{s}$ is the traction vector on a side of the interface with unit normal $\mathbf{n}$ and $v$ is the (normalized) normal component of displacement jump across the interface, generally dependent on an interface coordinate. The interface constitutive quantities $\sigma_{\max}$, $\delta$ characterize the interface

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[1] The term *crack* is reserved for the static sharp crack of fracture mechanics.

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http://dx.doi.org/10.1016/j.ijsolstr.2012.12.024

strength and the dimensionless force length, respectively. Interface nonuniformities including crack-like defects are considered by allowing the interface strength to be a function of interface coordinate $x$, i.e., $\sigma_{\max}(x)$ (Needleman, 1990a; Needleman 1990b).

The approach taken here for the analysis of the orthotropic cohesive fracture problem is similar in some respects to one that is used in the well developed theory of *static sharp cracks* in plane rectangular anisotropic media in general, and orthotropic media in particular. In these problems, stress intensity factors for straight cracks in a variety of geometrical and loading configurations can be obtained by means of orthotropy rescaling of the governing orthotropic elasticity equations resulting in problems with cubic symmetry or, isotropic symmetry (provided the unscaled problem is *degenerate orthotropic*, i.e., $\rho = 1$, $\rho$ is the orthotropic shear parameter (Suo, 1990; Suo et al, 1991)). The argument follows that, because many problems of technical interest have been solved for the simpler symmetry classes, their solutions can be exploited to yield the desired fully orthotropic (or degenerate orthotropic) solutions without much additional effort. Although the essence of the nonlinear cohesive fracture problem is fundamentally different from that of the linear static sharp crack, the overall philosophy employed here is the same as that used in Suo (1990), Suo et al (1991) to treat sharp cracks, i.e., to employ a rescaling of the equations to extract solutions for orthotropic media from isotropic or cubic media. Because the cohesive fracture problem, in contrast to the sharp crack problem, involves a nonlinear displacement boundary condition, the general form of rescaling introduced by Krenk (1979) will be employed. For problems with degenerate orthotropy, it is shown that rescaling reduces the problem to isotropic symmetry while fully orthotropic problems are reduced to problems with cubic symmetry. In the former case, isotropic solutions obtained by Nguyen and Levy (2009, 2011) are used to directly obtain orthotropic response via rescaling. In the later case an exact methodology, based on elasticity solutions for problems of cubic symmetry, is developed for loading consisting of pointwise prescribed strong boundary conditions on the upper and lower layer surfaces, and resultant prescribed weak boundary conditions on the side surfaces. This system models cohesive fracture in a single layer under a wide range of loading conditions. In particular, the stress function equation is solved in two sub layers adhered to each other along a cohesive interface and exact elasticity solutions for the boundary displacement components are written for each sub-layer. These are then pieced together to form integral equations governing displacement discontinuity components normal and tangent to the interface. The equations are necessarily nonlinear owing to nonlinear interface traction-separation/slip relations required to characterize the interface. The solution process proceeds by using eigenfunction expansion methods to reduce the integral equations to an infinite set of nonlinear algebraic equations which are then truncated and solved numerically.

In the next section (Section 2) orthotropy rescaling of the elasticity equations is discussed and extended to include the nonlinear cohesive interface boundary condition. Two problems involving degenerate orthotropic media are then solved by a rescaling of isotropic solutions obtained previously by the authors (Nguyen and Levy, 2009, 2011). The first problem deals with a cohesive defect (nonuniformity in interface strength) in a layer for which there is symmetry about the defect line (Fig. 1), while the second deals with a problem of stability of interfacial separation in a trilayer system (Fig. 5). Because fully orthotropic solutions can be obtained from cubic symmetry solutions, Section 3 presents an exact general theory of nonlinear cohesive defect growth in a layer composed of cubically symmetric media. Explicit results, including a discussion of the issue of $\rho$ dependence, are presented for an edge notch bend configuration. The section closes with a demonstration of the remarkable fact that, for this configuration, increasing the orthotropic stiffness ratio can precipitate a transition from more or less ductile defect growth to a quasi-brittle type of response characterized by an abrupt jump in defect length. The final section (Section 4) summarizes the findings and suggests further extensions of the work.

## 2. Orthotropy rescaling; degenerate orthotropic solutions

### 2.1. Displacement boundary value problem

Hooke's law for plane orthotropic linear elastic media assumes the form (Lekhnitskii, 1981),

$$e_i = \beta_{ij} t_j, \quad i,j = 1,2,6 \tag{1}$$

where the strain components $e_i$, $i = 1, 2, 6$ are given in terms of the strain tensor components by $[e_1, e_2, e_6]^T = [\varepsilon_{xx}, \varepsilon_{yy}, 2\varepsilon_{xy}]^T$ and the stress components $t_i$, $i = 1, 2, 6$ are given in terms of the stress tensor components by $[t_1, t_2, t_6]^T = [\sigma_{xx}, \sigma_{yy}, \sigma_{xy}]^T$. The coefficients $\beta_{ij}$, $i, j = 1, 2, 6$ have different forms depending on whether one is dealing with plane strain or plane stress. The coefficients $\beta_{ij}$, $i, j = 1, 2, 6$ have different forms depending on whether one is dealing with plane strain or plane stress. Thus, if $\beta_{ij} = \alpha_{ij}$, $i, j = 1, 2, 6$ are plane stress components, then the plane strain components are given by $\beta_{ij} = \alpha_{ij} - \alpha_{i3}\alpha_{j3}/\alpha_{33}$, $i, j = 1, 2, 6$. In terms of engineering moduli the $\alpha_{ij}$ are defined by, $\alpha_{11} = 1/E_1$, $\alpha_{22} = 1/E_2$, $\alpha_{12} = -\upsilon_{21}/E_2$, $\alpha_{21} = -\upsilon_{12}/E_1$, $\alpha_{66} = 1/G$ and $\alpha_{13} = -\upsilon_{31}/E_3$, $\alpha_{23} = -\upsilon_{32}/E_3$, $\alpha_{33} = 1/E_3$ with $\alpha_{12} = \alpha_{21} = -\upsilon_{21}/E_2 = -\upsilon_{12}/E_1$ (all other coefficients zero). The quantities $(E_1, E_2, E_3)$ are stiffnesses, $G$ is the in-plane shear modulus, and $(\upsilon_{12}, \upsilon_{21}, \upsilon_{13}, \upsilon_{31}, \upsilon_{23}, \upsilon_{32})$ are Poisson ratios, i.e., $\upsilon_{ij}$ characterizes the contraction in the $j$ direction due to an extension in the $i$ direction. There are four independent constants, i.e., five constants $(\beta_{11}, \beta_{22}, \beta_{12}, \beta_{21}, \beta_{66})$ connected by one constraint $\beta_{12} = \beta_{21}$. Following Krenk (1979) introduce quantities valid for both plane stress and plane strain: the *effective stiffness* $E$, the *effective Poisson ratio* $\upsilon$, the *shear parameter* $\rho$ and the *stiffness ratio* $\lambda$,[2]

$$E = \sqrt[-1]{\beta_{11}\beta_{22}}, \quad \upsilon = \sqrt{\frac{\beta_{12}\beta_{21}}{\beta_{11}\beta_{22}}}, \quad \rho = \frac{1}{2}\frac{2\beta_{12} + \beta_{66}}{\sqrt{\beta_{11}\beta_{22}}}, \quad \lambda = \frac{\beta_{11}}{\beta_{22}} \tag{2}$$

It is well known that, for rectilinear anisotropic media, the equilibrium equations will be satisfied when the stress components are written in the form $\sigma_{xx} = \frac{\partial^2\varphi}{\partial y^2}$, $\sigma_{yy} = \frac{\partial^2\varphi}{\partial x^2}$, $\sigma_{xy} = -\frac{\partial^2\varphi}{\partial x\partial y}$ and the compatibility equations will be satisfied when the stress function $\varphi$ satisfies the differential equation $\frac{\partial^4\varphi}{\partial x^4} + 2\rho\lambda^{1/2}\frac{\partial^4\varphi}{\partial x^2\partial y^2} + \lambda\frac{\partial^4\varphi}{\partial y^4} = 0$ where $\lambda, \rho$ are defined above (Lekhnitskii, 1981).

If a change of variables[3] (Krenk, 1979) is introduced according to,

$$\begin{aligned}
&\hat{x} = \lambda^{1/8}x, \quad \hat{y} = \lambda^{-1/8}y, \quad \hat{u}_x = \lambda^{-1/8}u_x, \quad \hat{u}_y = \lambda^{1/8}u_y, \\
&\hat{\varepsilon}_{xx} = \lambda^{-1/4}\varepsilon_{xx}, \quad \hat{\varepsilon}_{yy} = \lambda^{1/4}\varepsilon_{yy}, \quad \hat{\varepsilon}_{xy} = \varepsilon_{xy}, \\
&\hat{\sigma}_{xx} = \lambda^{1/4}\sigma_{xx}, \quad \hat{\sigma}_{yy} = \lambda^{-1/4}\sigma_{yy}, \quad \hat{\sigma}_{xy} = \sigma_{xy},
\end{aligned} \tag{3}$$

then the following standard relationships are true,

$$\begin{aligned}
&\hat{\sigma}_{xx} = \frac{\partial^2\hat{\varphi}}{\partial\hat{y}^2}, \quad \hat{\sigma}_{yy} = \frac{\partial^2\hat{\varphi}}{\partial\hat{x}^2}, \quad \hat{\sigma}_{xy} = -\frac{\partial^2\hat{\varphi}}{\partial\hat{x}\partial\hat{y}}, \\
&\hat{\varepsilon}_{xx} = \frac{\partial\hat{u}_x}{\partial\hat{x}}, \quad \hat{\varepsilon}_{yy} = \frac{\partial\hat{u}_y}{\partial\hat{y}}, \quad 2\hat{\varepsilon}_{xy} = \frac{\partial\hat{u}_y}{\partial\hat{x}} + \frac{\partial\hat{u}_x}{\partial\hat{y}}.
\end{aligned} \tag{4}$$

The stress function $\hat{\varphi}$ now satisfies the rescaled equation,

$$\frac{\partial^4\hat{\varphi}}{\partial\hat{x}^4} + 2\rho\frac{\partial^4\hat{\varphi}}{\partial\hat{x}^2\partial\hat{y}^2} + \frac{\partial^4\hat{\varphi}}{\partial\hat{y}^4} = 0, \tag{5}$$

[2] For many materials of interest $0 < \rho < 5$, $1/20 < \lambda < 20$ (Bao et al, 1992).

[3] In what follows a caret will designate a scaled variable.