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Constitutive model for monotonic and cyclic responses of loosely cemented sand formations

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ABSTRACT

This paper presents a model to simulate the monotonic and cyclic behaviours of weakly cemented sands. An elastoplastic constitutive model within the framework of bounding surface plasticity theory is adopted to predict the mechanical behaviour of soft sandstone under monotonic and cyclic loadings. In this model, the loading surface always passes through the current stress state regardless of the type of loading. Destruction of the cementation bonds by plastic deformation in the model is considered as the primary mechanism responsible for the mechanical degradation of loosely cemented sands/weak rock. To model cyclic response, the unloading plastic and elastic moduli are formulated based on the loading/reloading plastic and elastic moduli. The proposed model was implemented in FLAC2D and evaluated against laboratory triaxial tests under monotonic and cyclic loadings, and the model results agreed well with the experimental observations. For cyclic tests, hysteresis loops are captured with reasonable accuracy.

1. Introduction

This paper focuses on the constitutive model and degradation behaviour of cemented sand/soft sandstone under monotonic and cyclic loadings. Two main approaches for the study of weak sandstone degradation behaviour caused by repeated loading include laboratory testing by conducting cyclic loading tests and the development of cyclic plasticity theories. The majority of the studies in the area of cyclic loading of sandstone or cemented sand are limited to earthquake (dynamic) type of loading. Few studies have characterized the deformation properties of sandstone under slow cyclic loading both theoretically and experimentally. In general, the behaviour of geomaterials under cyclic loading is remarkably complex. This may be due to the dependence of the constitutive relationship on pressure and void ratio as well as the nonlinear behaviour of the sand matrix (Russell and Khalili, 2004; Khalili et al., 2005, 2006). Even the most sophisticated models cannot provide accurate predictions under general cyclic loading (O'Reilly and Brown, 1991). Therefore, development of a reliable model to capture the cyclic behaviour of geomaterials has become one of the most challenging issues in constitutive modelling (Vermeer and de Borst, 1984).

Most models for slow cyclic loading have been proposed for cohesionless soils and few studies have been conducted on sandstone behaviour in response to slow cyclic loading. Thus a model that could capture soft sandstone response under slow cyclic loading would be a significant advancement. The particular difficulty in the integration of the critical state concept in cyclic modelling is noted in the literature (Imam and Chan, 2008). In this paper, a critical state constitutive model is presented for slow cyclic loadings for soft sandstone.

The traditional plasticity theory appears to be unsuitable for modelling cyclic loading since it predicts a purely elastic response during unloading and reloading within the yield surface. That is, no plastic deformation is predicted for unloading and reloading unless the stress path reaches the yield surface again (Chen and Han, 2007). This is not suitable for modelling cyclic loading because, in reality, all unload-reload cycles result in the gradual accumulation of plastic strain and energy dissipation (Khong, 2004; Lenart, 2008) as shown schematically in Fig. 1. In other words, the response in

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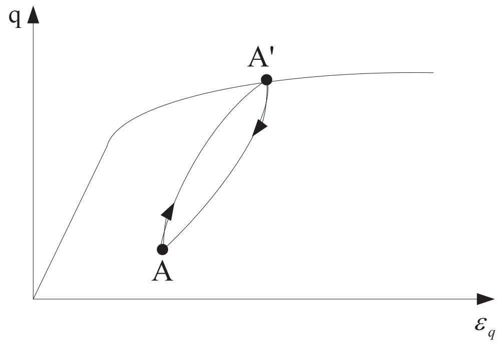

Fig. 1. Unloading and reloading from an elastoplastic state: perfect hysteresis loop in a complete cycle.

Fig. 1 suggests that the loading and unloading stress paths are not the same. This is known as hysteresis, and it shows that the material fails to recover all the energy it receives in the loading–unloading cycle (O'Reilly and Brown, 1991). This is attributed to energy dissipation due to plastic deformation (Lenart, 2008). Hysteresis is the result of non-uniform deformation of the material in which different parts of the material are undergoing different stages of loading and unloading.

The effect of non-uniform deformation at different stages of loading and unloading can be illustrated using a friction block model. Uniform deformation is analogous to a single block as shown in Fig. 2. In this case, there is only one displacement in the system, which is represented by u. There is no slipping until the horizontal force (T) reaches the maximum frictional force (F_k) when the block starts to move. At some displacement (u_1), if T decreases below F_k, movement will cease immediately and the force will vary between zero and F_k. There is no movement until T reaches F_k again and the displacement will continue from u_1. There is no hysteresis effect, and there is no study reported during the unloading and reloading cycle at u_1.

Non-uniform deformation in a material can be conceptually represented by a series of blocks as shown in Fig. 3. In this case, four blocks are connected by three springs with stiffness (K), and each block also is subjected to a normal force (N). The horizontal force (T) is slowly increased until the first block on the right, block 4, starts to move. There is no horizontal force applied on the other blocks until the spring between blocks 4 and 3 starts to compress. The force that will transmit to block 3 will be equal to the difference between T and F_k, where F_k is the frictional resistance at the base of each block. Again, there is no force applied on block 2 until the force in the spring between blocks 4 and 3 exceeds F_k in block 3. The process continues until all of the blocks start to move when T is equal to or exceeds $4F_k$. Since the movement of all the blocks is in the direction of T, F_k will be acting in the opposite direction (Fig. 3a).

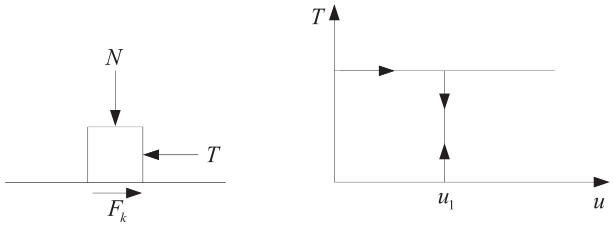

Fig. 2. Single frictional block on a flat surface and corresponding force-displacement response.

If T decreases below $4F_k$ after some movement of the blocks, the frictional force on block 4 will start to decrease until the direction is reversed as shown in Fig. 3b. There is no movement of block 4 until T decreases below $2F_k$. In this case, the force in the spring between blocks 4 and 3 will also decrease until the direction of the frictional force under block 2 reverses its direction. In this case, T will become zero, representing a fully unloaded state as shown in Fig. 3c.

If T increases again, there is no movement in block 4 until its frictional force changes its direction and the value of T is equal to $2F_k$ as shown in Fig. 3d. This represents that the reloading stage after T has been fully unloaded. It is clear that the reloading path is different from the unloading path since the mobilization of the frictional force under the blocks is different, unlike the case of a single block. Movement of block 4 occurs when T increases above $2F_k$. When T is equal to $4F_k$, the frictional forces under all of the blocks point in the same direction and movement will continue in the direction of T. Fig. 4 shows force-displacement response of block 4.

As demonstrated in this simple system of blocks, the hysteresis effect is a result of non-uniform mobilization of frictional forces under the blocks since they are connected by deformable springs. If the blocks are connected by rigid springs, the hysteresis effect will disappear.

In the case of a real material, since the stresses and strains in the material are generally non-uniform at the mesoscopic scale, it will give rise to the hysteresis effect much like the series of blocks connected by deformable springs. Therefore, energy dissipation occurs during the unloading and reloading process below the latest yield point.

The shortcomings of the classical plasticity theory led to extensive research, beginning in the 1960s, on developing more sophisticated plasticity models to capture the cyclic behaviour of geomaterials (Yu, 2006). Advanced constitutive models that have been introduced within the plasticity framework include multi-surface plasticity (Iwan, 1967; Mroz, 1967; Mroz et al., 1978, 1979), bounding surface plasticity (Dafalias and Popov, 1975; Krieg, 1975; Dafalias, 1982, 1986; Bardet, 1986; Khong, 2004; Khalili et al., 2005, 2006; Yang et al., 2011), generalized plasticity (Zienkiewicz et al., 1985; Pastor et al., 1985, 1990; Ling and Yang, 2006; Chung, 2010), and subloading surface plasticity (Hashiguchi, 1989; Hashiguchi and Chen, 1998).

Note that for our target material (i.e. naturally or artificially cemented sand), few constitutive models have been developed to simulate cyclic behaviour. For instance, Weng and Ling (2012) and Weng (2014) proposed their constitutive models based on generalized plasticity theory and verified their models against laboratory one-way cyclic loading of sandstone. Tariq and Maki (2012) conducted one-way cyclic loading tests on artificially cemented sand. However, they did not develop any constitutive model to simulate their experimental observations. Zhang et al. (2013) conducted one-way cyclic loading tests on a specific sandstone known as red sandstone. However, their proposed elastoplastic constitutive model was not verified against cyclic shear stress tests. Fu et al. (2014) developed a constitutive model within the framework of generalized plasticity and verified their model against experimental observations of one-way cyclic loading of rufous sandstone. Liu et al. (2016) conducted several laboratory one-way cyclic loading tests on structured (i.e. cemented) soils. However, they did not suggest any constitutive model to simulate experimental results.

The aim of this paper is to present a continuum elastoplastic constitutive model within the framework of bounding surface plasticity to model cyclic loading of frictional and cohesive material. A critical state constitutive model proposed by Imam (1999) and

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