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Three dimensional axisymmetric sound propagation through a duct with aerosol

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ABSTRACT

The aim of this study is to develop solutions for the propagation of spinning sound waves in air-filled circular ducts containing water droplets (regarded as an aerosol). There exist several theoretical approaches in literature for treatment of basic (mostly 1-D) aerosol problems with certain limitations, but these approaches are mostly suitable for plane waves. A numerical methodology based on the linearized Euler equations in frequency domain has been developed to account for spatial effects both in azimuthal and radial directions. The interactions between the fluid and particle phases are considered through momentum. The paper also presents an analytical solution with which the numerical solutions are compared. Example solutions for plane and spinning modes are presented with emphasis on the effects of aerosol parameters on propagation characteristics as well as geometric cut-off frequencies.

1. Introduction

Study of sound propagation in an aerosol has been focus of interest for many applications. These include analysis and measurement of cloud and humidity parameters (atmospheric sciences), control of chemical processes (chemical engineering), agglomeration of dust and particles in air (environmental engineering), suppression of instabilities in combustion chambers (propulsion applications), and noise reduction (aerospace and marine applications). Many researchers have theoretically dealt with this topic, developing new approaches particularly to improve the models for the interactions between gas (host medium) and aerosol particles in order to account for actual physics of the system [1–13] Simultaneously reported were dedicated experimental studies measuring acoustic parameters like acoustic absorption and dispersion [6,14–18].

Temkin and Dobbins [6] developed a coupled-phase theory based on particulate relaxation process and continuum approach. The relation was investigated between particulate relaxation and maximum attenuation per wavelength. The validity of their theory was tested by comparing the predictions with the experimental results [6]. Marble and Wooten [7] extended the theory of Temkin and Dobbins' to include effects of evaporation and condensation on attenuation and dispersion. Similar study was also reported by Cole and Dobbin's [8]. Davidson [9] attempted to improve the theory of Cole and Dobbin's [8] in order to reduce the mismatch between the theory and measurements. Gubaidullin [10] studied the case when the wave was like plane, spherical and cylindrical disturbances and the medium was a gas-vapor-drop mixture. He tracked the evolution of the waveform in time by solving the system of equations numerically. In another study, he took into account a second particle phase [11]. Kandula [12,13] treated this subject by including the non-linear effects of relaxation processes and transpiration

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effects on droplet evaporation for the cases of high Reynolds numbers. As summarized above, most of the studies on the subject have been performed primarily analytically and experimentally, whereas very few studies are related to numerical approaches.

On the practical front; engineers have employed aerosol acoustics to address technical problems in various fields. One important application is the combustion instability in thrust chambers due to coupling of flame dynamics with acoustic perturbations. In addition to the investigation of acoustic modes of the chamber, the effect of particles like aluminum with regard to damping of high pressure amplitudes has been studied in literature [19–23]. Another application is related to reduction of air pollution due to fine particles by means of acoustic agglomeration by generating a sound field in after-treatment systems [24–27].

It is clear that interaction mechanisms between acoustic waves and the environment they propagate through may vary. For sound propagation through ducts, these interactions are with the confining walls, rigid or non-rigid, and perhaps with some contained flow non-uniformities, laminar or turbulent, and to some special extent with suspended droplets as in the present paper. Effects of all these will be to different scales, and treating all of them in a single simulation is most often prohibitive – if not unnecessary. Therefore, simplifications are often made. Although satisfactory efficacy has been obtained in some engineering applications using 1-D approach [31], both analytically and numerically, the effect of particles on sound wave propagation in 3-D ducts has not been investigated. On the other hand, any wave with a non-zero mode order, possibly spinning, which is allowed only in 3-D treatment, has a wave front which has a normal deviating from the axis of the duct, and the effective propagation length over the duct length is different than that of a plane wave at the same frequency. Hence, the damping characteristics by aerosol may be expected to be quite different. Consideration of a theoretical solution in this respect may provide a significant understanding of sound attenuation and dispersion in such systems.

The aim of the present paper is to solve the propagation of spinning sound waves through infinitely long circular ducts containing an aerosol. The scope of the treatment is limited only to the effects of suspended droplets (aerosol) on acoustic propagation through axisymmetric ducts. Then, a 3-D axisymmetric, frequency domain formulation of the linear governing equations both for the gas and particle phases is employed and discretized using 4th-order finite differences. In addition, the reference, 1-D analytical solution given by Temkin [28] is extended to 3-D axisymmetric case with hard wall boundary condition. Then, the validity of the approach is shown by comparing the computed numerical results with the analytically obtained ones. The effect of particle parameters on the acoustic properties of the duct is also discussed using the analytically extended solutions.

2. Methodology

As aforementioned, the goal of this study is to solve propagation of spinning sound waves through infinitely long circular ducts containing an aerosol. The study includes both numerical and analytical development. Due to its superiority for aerosol acoustics the coupled phase theory [28] has been chosen for the present purposes. The following assumptions are made: Each phase occupies a separate volume; the physical dimensions of particles are acoustically small; total number of the particles is sufficient for use of the continuum approach; average quantities are suitable to represent the physical properties; physical collisions and scattering effects among particles are ignored, but their interactions with the other phase elements are included through simple momentum exchange. These assumptions are satisfied in general when the medium is a dilute aerosol.

2.1. Numerical approach

In the numerical approach, the infinite circular duct solution domain is truncated to a finite size with sound absorbing regions called Perfectly Matched Layers (PML) at the two ends, as depicted in Fig. 1. The frequency domain linearized Euler equations (LEE) are solved in the entire domain with addition of damping terms in the absorbing regions. Sound waves are introduced into the solution domain from one end of the duct. The governing equations, boundary conditions, and the form of incident sound waves are given in the following sub sections.

2.1.1. Governing equations

Assuming the acoustic field is axisymmetric (due to axisymmetric geometry, as well as time harmonic, perturbation variables in the duct may be written in the form,

$$\mathbf{q}'(\mathbf{x},t) = \Re\{\widehat{\mathbf{q}}(x,r,\omega)\exp(im\theta - i\omega t)\} \tag{1}$$

where ω is the angular velocity, and m is an integer called the azimuthal mode order. The complex form of the dependent variables governing the linearized equations are given as

$$\widehat{\mathbf{q}} = [\widehat{\rho}_g, \widehat{u}_g, \widehat{v}_g, \widehat{w}_g, \widehat{p}_g, \widehat{\phi}, \widehat{u}_p, \widehat{v}_p, \widehat{w}_p]^T \tag{2}$$

where $\widehat{\rho}_g$ is the gas (air) density perturbation, $\widehat{u}_g$, $\widehat{v}_g$, $\widehat{w}_g$ are the gas axial, radial, and azimuthal velocity perturbations, respectively, $\widehat{p}_g$ is the gas pressure perturbation, while $\widehat{\phi}$ is the particle (water droplets) volume concentration perturbation, $\widehat{u}_p$, $\widehat{v}_p$, $\widehat{w}_p$ are the particle axial, radial, and azimuthal velocity perturbations, respectively. In the present approach