Journal of Sound and Vibration 432 (2018) 50–64

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Active vibration suppression of a novel airfoil model with fractional order viscoelastic constitutive relationship

Qi Liu [a], Yong Xu [a,b,c,*], Jürgen Kurths [b,c]

[a] Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an, 710072, China
[b] Potsdam Institute for Climate Impact Research, Potsdam, 14412, Germany
[c] Department of Physics, Humboldt University Berlin, Berlin, 12489, Germany

A R T I C L E I N F O

Article history:
Received 11 February 2018
Revised 16 May 2018
Accepted 10 June 2018
Available online XXX
Handling Editor: L. N. Virgin

Keywords:
Viscoelastic airfoil model
Fractional order
Averaging technique
Amplitude-frequency relation
Sliding mode control
Vibration suppression

A B S T R A C T

This paper aims to investigate the active vibration suppression of a fractional two-degree-of-freedom viscoelastic airfoil (TDOFVA) model with a harmonic external force by means of the sliding mode control (SMC) scheme. The viscoelastic behavior is described as a fractional-order derivative, leading to a new fractional TDOFVA model. Subsequently, an averaging technique is extended to derive the amplitude-frequency relations, and its correctness is verified by Monte Carlo simulations. In addition, effects of the system parameters on the dynamics are explored. To achieve a vibration suppression, we convert the TDOFVA system into a series of fractional-order differential equations. Then, a SMC strategy is employed, in which a fractional-order integral sliding surface is presented and asymptotical stability analysis of the SMC is performed. Several numerical results are presented to illustrate the performances of the proposed SMC scheme, which indicate that the given SMC methodology is effective to realize a vibration suppression of the TDOFVA model.

1. Introduction

Viscoelastic materials, such as soft soil, brain tissue and colloid, can exhibit a mechanical behavior between pure elasticity and pure viscosity, which have been widely applied in different fields [1–6]. An appropriate model describing the viscoelastic materials is essential to the study and accurate design incorporating the particular material. Previously, the viscoelastic behavior was usually characterized by integro-differential equations, and several classical integer-order models to describe viscoelastic behavior have been proposed, including the Maxwell model, the Kevin model, the Voigt model, etc. [1]. To achieve a higher fitting precision with the experimental data, more spring or dashpot elements are often combined in the classical integer-order models [1].

Over the past few decades, the fractional calculus has been successfully employed in modeling the constitutive relations of viscoelastic behaviors [7–18]. Several fractional-order viscoelastic models have been established and validated by experimental data, such as the fractional Poynting-Thomson model [11], the fractional Maxwell model [13], and the fractional Kelvin-Voigt model [14]. In fact, these fractional-order viscoelastic models can be viewed as a natural generalization of the classical integer-order versions. The fractional-order ones can reflect memory effects much more accurately, which have been simulated in various studies [7–18]. However, the effects of the fractional-order viscoelastic property on an aeroelastic wing system have not been well addressed. Recently, dynamical behaviors of an airfoil model involving a fractional-order viscoelasticity were

* Corresponding author. Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an, 710072, China.
E-mail address: hsux3@nwpu.edu.cn (Y. Xu).

https://doi.org/10.1016/j.jsv.2018.06.022
0022-460X/

studied [19], but in which harmonic excitation, theoretical analysis and its related control problems have not been considered yet. A two-dimensional airfoil section, as a simplified model, can be employed to obtain/reveal several dynamical behaviors of airfoil in practical flight [20]. In addition, the external loading imposing on the airfoil system could be periodic in real engineering applications, such as aircraft engines. Therefore, in the present work, the viscoelasticity of airfoil materials is described by fractional-order derivative, then a novel two-degree-of-freedom viscoelastic airfoil (TDOFVA) model under a harmonic excitation is presented.

In recent years, several methods have been developed to analyze the dynamical behaviors of systems with fractional-order derivatives, including numerical methods [12,19,21] and approximately analytical techniques [10,16–18,22–25]. The theoretical research, to analyze the effects of system parameters, is extremely significant in fractional-order dynamical systems. For example, Xu et al. studied the dynamical responses of the fractional-order Duffing system with a random disturbance through combining the Lindstedt-Poincaré method and the multiple-scale approach [17,22,23]. Shen et al. investigated several linear and nonlinear dynamical systems with a fractional-order derivative via the averaging method [24,25]. The TDOFVA model, as we know, can be regarded as two coupled Duffing equations. Thus, several techniques for the single-degree-of-freedom Duffing system with a fractional-order derivative [22–24] can be employed on the proposed coupled TDOFVA model, and the averaging method [24,25] is extended in this paper.

As a typical nonlinear self-excited system, the aeroelastic airfoil system may possess a variety of nonlinear responses, including limit cycle oscillations (LCOs), jump phenomenon, bifurcation and even chaotic motion [26]. For certain wing store configurations, a fighter aircraft undergoes undesirable LCOs, which can degrade the flight performance of an aircraft and even can cause catastrophic consequences [27]. Hence, active vibration suppressions become increasingly important to ensure the safety and efficiency of the aircraft, which is a very challenging field and have attracted the interest of researchers in aerospace and control communities. Nevertheless, the vibration suppression for the viscoelastic airfoil system, in particular, with a fractional order viscoelastic constitutive relationship, has not been well examined yet which is the main objective of this paper.

Many active control techniques, such as feedback control [28,29], adaptive control [30], and sliding mode control (SMC) [31–33], have been successfully used to design controllers for the vibration suppression. SMC, as an elegant and powerful tool, has been applied in different linear and nonlinear systems [34]. However, compared with the conventional integer-order version, the fractional-order SMC can provide good performances on reducing the chattering phenomenon, which has stimulated the interest of a number of scholars [35–37]. For instance, Xu et al. were devoted to investigating the control of a class of fractional chaotic systems with parameter perturbations by the SMC method [37]. In fact, introducing some additional state variables, the TDOFVA system can be converted into a series of fractional-order differential equations. Then, the control of the TDOFVA model can be regarded as a fractional-order control problem, which are different from the previous studies for the integer-order case. Consequently, a fractional-order SMC is proposed to obtain an effective controller for suppressing vibrations in this study.

This paper is organized as follows. In Section 2, a novel fractional TDOFVA model subject to harmonic excitation is presented. In Section 3, amplitude-frequency equations of the proposed TDOFVA model are derived by extending the averaging technique [24,25], and its correctness is confirmed through Monte Carlo simulations. In addition, effects of the system parameters on the amplitude-frequency responses are examined, and a jump phenomenon between larger amplitude LCOs and smaller amplitude ones is observed. The larger amplitude LCOs are highly undesirable as they can affect ride discomfort and might lead to catastrophic failures, and hence need to be suppressed. In Section 4, the active vibration suppression of the TDOFVA system is achieved via a fractional-order SMC strategy, and numerical simulations are carried out to demonstrate the effectiveness and feasibility of the proposed control scheme. The conclusions are presented to close this paper in Section 5.

2. The fractional TDOFVA model

2.1. Preliminaries

We start with basic definitions and properties of fractional calculus. So far, several definitions of fractional-order derivative have been given, and the Riemann-Liouville and Caputo definitions are the two most commonly used. However, the Caputo definition owns the same initial conditions as the classical integer-order version, which has more applications in physics and engineering [7]. Therefore, the Caputo definition is adopted here.

Definition 1. [7] The Riemann-Liouville fractional-order integral operator of order p for a continuously differentiable function $f(t)$ is defined by

$$ {}_0I_t^p f(t) = \frac{1}{\Gamma(p)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-p}} d\tau,\ t > 0,\ p > 0, $$

where $\Gamma(\cdot)$ is Euler's Gamma function.

Definition 2. [7] The Riemann-Liouville fractional-order derivative of order p for a continuously differentiable function $f(t)$ is given by

$$ {}_0^{RL}D_t^p f(t) = \frac{d^n}{dt^n} \left({}_0I_t^{n-p} f(t) \right) $$