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Resonances and vibrations in an elevator cable system due to boundary sway

Nick V. Gaiko [*], Wim T. van Horssen

Department of Mathematical Physics, Delft Institute of Applied Mathematics, Delft University of Technology, Mekelweg 4, Delft, 2628 CD, The Netherlands

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ABSTRACT

In this paper, an analytical method is presented to study an initial-boundary value problem describing the transverse displacements of a vertically moving beam under boundary excitation. The length of the beam is linearly varying in time, i.e., the axial, vertical velocity of the beam is assumed to be constant. The bending stiffness of the beam is assumed to be small. This problem may be regarded as a model describing the lateral vibrations of an elevator cable excited at its boundaries by the wind-induced building sway. Slow variation of the cable length leads to a singular perturbation problem which is expressed in slowly changing, time-dependent coefficients in the governing differential equation. By providing an interior layer analysis, infinitely many resonance manifolds are detected. Further, the initial-boundary value problem is studied in detail using a three-timescales perturbation method. The constructed formal approximations of the solutions are in agreement with the numerical results.

1. Introduction

Within the last decade, high-rise buildings have entered a new era of “megatall” buildings, which are over 600 m in height. The construction of such tall buildings has many practical limitations due to various issues. The higher buildings rise, the more vulnerable they become to wind influence. This wind-force can lead to building sway, which can initiate the motion of elevator cables. Resonances in elevator cables can damage shaft devices or cause entanglements in the shaft. In fact, internal transportation systems play a crucial role in the building functionality. That is why considerable attention should be paid to improvement of elevator technologies to prevent any damage, and consequently downtime of elevators. However, the increasing complexity of the engineering structures increases the complexity of their analysis. Therefore, it is also important to develop advanced analytical models in order to tackle this complexity; one of which is presented in this paper.

This work is an extension of the study by Sandilo and van Horssen [1], where the lateral vibrations of an elevator cable system with a small sinusoidal excitation at its upper end was studied. The results showed that $\mathcal{O}(\epsilon)$ excitation at the upper end of the cable resulted in $\mathcal{O}(\sqrt{\epsilon})$ autoresonance responses. In contrast to that work, a mathematical model developed in the current paper is made closer to reality. One of the reasons is that the formulation of the problem includes bending stiffness of the cable allowing to obtain more accurate results for higher-order frequencies. The other reason is that both boundaries of the cable are excited by a harmonic function representing wind-induced sway of the building. In reality, when the building is acted upon by high velocity winds, it tends to sway in the lateral direction. This lateral motion translates into lateral motion of the cable. Note

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[*] Corresponding author.
E-mail address: N.Gaiko@tudelft.nl (N.V. Gaiko).

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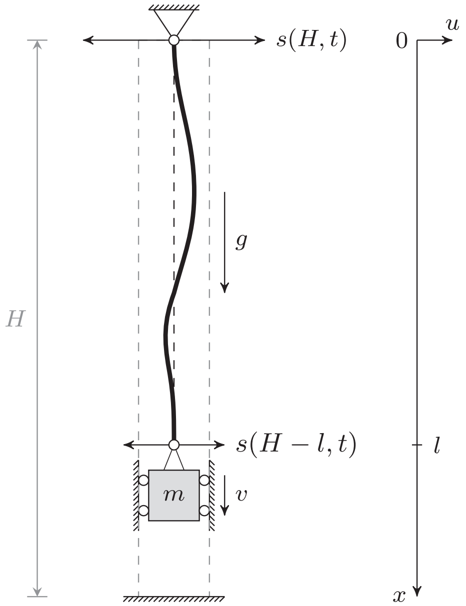

Fig. 1. Schematic of a vertically moving cable with an attached elevator car at the lower end in a swaying building.

that in our mathematical model the sway related harmonic function changes with the travel height of the elevator.

A lot of other research has been conducted on similar types of problems. Kaczmarczyk [2] analyzed resonance in a catenary-vertical cable with slowly varying length under a periodic external excitation. Zhu and Ni [3] investigated a class of axially moving continua with arbitrarily varying length. Zhu and Xu [4] studied the dynamics of elevator cables with small bending stiffness. Zhu and Teppo [5] developed a new scaled model describing the lateral vibrations of an elevator cable with a variable length for a high-rise, high-speed elevator. Kaczmarczyk and Ostachowicz derived a mathematical model [6] and provided a numerical simulation of the dynamic response [7] for transient vibrations in deep mine hoisting cables. Zhu and Chen [8] presented a control method to dissipate the vibratory energy of the cable. Moreover, the authors introduced a new experimental method to validate the theoretical results for the (un) controlled lateral vibrations. Kimura et al. [9] studied forced vibrations of an elevator rope with both ends excited by wind-induced displacement sway of the building. Kaczmarczyk [10] developed a model describing the lateral dynamics of long vertically moving ropes for high-rise transportation. Crespo et al. [11] investigated nonlinear responses of an elevator rope system coupled with the elevator car sheave motion. Bao et al. [12] studied the nonlinear response of a flexible hoisting rope with time-varying length. Gaiko and van Horssen [13] considered lateral vibrations of a vertically moving string with in time harmonically varying length.

In this paper we study, in particular, the lateral vibrations of a vertically moving beam (with linearly in time varying length) excited at both boundaries by a harmonic function in the horizontal direction (see Fig. 1). From the physical point of view, the motivation of this work is described as follows. When the fundamental frequency of the building sway matches one of the natural frequencies of elevator cable oscillations, then resonance emerges. This match happens due to a slow variation of the cable's length. In order to describe this phenomenon, an analytical methodology is developed in this paper. First, an internal layer analysis is provided to study the behavior of the solution in the neighborhood of resonance. To perform this analysis we introduce local variables in the vicinity of resonance and shift out of it on a value which follows from a certain balancing principle. Note that this value determines the size of the resonance interior layer. Next we proceed with a detailed three-timescales perturbation method. The crucial step in the construction of an approximation by this method is removing unbounded terms by providing the so-called secularity conditions. So, in order to obtain asymptotically valid approximations of the solution, one should distinguish between the behavior outside and inside resonance zones.

This paper is organized as follows. In Section 2 we make some assumptions and present an initial-boundary value problem describing the motion of the cable. Next, some transformations are introduced in order to simplify the construction of the approximation of the solution in Section 3. Further, we proceed with an internal layer analysis to study resonance in Section 4. Then, in Section 5 three-timescales are introduced to construct an accurate approximation of the solution on long timescales. Section 6 summarizes the results and provides some numerical experiments for the cable with small bending stiffness. Finally, in Section 7 we draw some conclusions based on both analytical and numerical results and also discuss future work.

2. Assumptions and mathematical model

In order to restrict the complexity of the analysis of the problem, it is necessary to make some assumptions: