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Stochastic resonance in a time-delayed feedback tristable system and its application in fault diagnosis

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Stochastic resonance (SR) phenomena in a time-delayed feedback tristable system driven by Gaussian white noise are investigated by simulating the potential function, mean first-passage time (MFPT), and signal-to-noise ratio (SNR) of the system. Through the use of a short delay time, the generalized potential function and stationary probability density function (PDF) are obtained. The delay feedback term has a significant effect on both equations, and that the parameters b, c, and d have different effects on the three wells of the potential function. The MFPT is calculated, which plays an extremely important role in research on particles escape rates. We find that the delay feedback term can affect the noise enhanced stability (NES). In addition, the SR characteristics are studied by the index of SNR. The simulation demonstrates that SNR is a non-monotonic distributed and that the peak SNR value can be attained by adjusting the appropriate parameters. Finally, the proposed theory is combined with a variable step method and applied to the detection of high frequencies in experiments. The result indicates that the fault frequency can be identified, and that the energy of the fault signal can be enhanced under suitable delay feedback parameters.

1. Introduction

For the sake of explaining the mechanisms of the ice age's periodic changes, Benzi et al. primitively proposed the concept of stochastic resonance(SR) in 1981 [1]. Up to now, SR has been used in fields such as meteorology, biology, and physics [2–8]. Recently, the study of SR in signal processing has received significant attention because it can be used to detect weak signals [9–12]. Since the beginning of the 21st century, the application of SR in mechanical fault diagnosis has been extensively studied [13–15]. In recent years, the dynamic behavior of time-delay SR systems has received much attention [16–18]. A number of studies have shown that noise and delay fundamentally affect system dynamics, and because the nonlinear

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interaction of the noise plays an important role in nonlinear systems, the delay time can lead to some new phenomena [19–22].

At present, most of the theories regarding SR involve traditional bistable systems. For example, Zhang et al. proposed an asymmetric delay bistable system and analyzed the influence of the asymmetric term and the delay term on the signal-to-noise ratio [16]. Gu investigated the phenomenon of time-delayed feedback in a bistable system with colored noise, and analyzed the physical characteristics of the delay term in the square term and the four-order term of the potential function [17]. Mei et al. studied the effect of time delay on an SR system with correlated and non-correlated multiplicative and additive noises [18]. Lu et al. [19] proposed a nonstationary weak signal detection strategy based on a time-delayed feedback stochastic resonance model and proved that this method was suitable for detecting signals with strong nonlinear and nonstationary properties. He et al. [20] addressed the problem of stochastic resonance in a time-delayed bistable system subjected to Gaussian white noise. Li et al. [21] proposed a novel weak signal detection method based on time-delayed feedback monostable stochastic resonance (TFMSR) system and adaptive minimum entropy deconvolution (MED) to realize the fault diagnosis of rolling bearings. Shi et al. [22] studied the dynamical complexity and stochastic resonance of a time-delayed asymmetric bistable system.

Compared with bistable systems, there are some studies on tristable stochastic resonance (TSR) systems. For instance, Lu et al. investigated a TSR system and adaptive adjustment of parameters using quantum-behaved particle swarm optimization, in order to obtain the optimal SNR and spectral output [23]. Shi et al. analyzed the influence of SNR gain on a TSR system driven by dichotomous noise and periodic signals [24]. In order to detect the signal characteristics of a multi-frequency weak signal, Han et al. designed a TSR model with wavelet transform and parameter compensation [25]. However, few studies have focused on the delays that occur in TSR systems, even though delays have great significance in nature and life. Therefore, it is necessary to study the effect of delays on TSR systems.

In this paper, a TSR system with time-delayed feedback driven by Gaussian white noise and periodic signals is proposed. Section 2 provides the principles of TSR and describes the time-delayed stationary probability function. In addition, their curves are discussed with regard to parameter changes. Section 3 investigates MFPT, and discusses the impact of delay feedback on changes in MFPT. In Section 4, the SNR formula is presented and then used to analyze the application of time-delay feedback stochastic resonance (TFTSR) in engineering. The effects of appropriate delay parameters on the amplitude of the fault characteristic frequency are positive. Finally, Section 5 provides the conclusion.

2. The tristable model driven by noises and time-delay feedback

The Langevin equation is used to describe the motion of “macroscopic” particles on a longer time scale. The Langevin equation for TSR can be written as follows [26]:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\mathrm{d}U(x)}{\mathrm{d}t} + s(t) + \eta(t) \tag{1}$$

where $s(t) = A\cos(2\pi ft)$ is the input periodic signal, in which A is the periodic signal amplitude, f is the driving frequency, and $\eta(t) = \sqrt{2D}\xi(t)$ is the Gaussian white noise item, in which D is the noise intensity and $\xi(t)$ represents Gaussian white noise with zero mean and unit variance.

For the tristable potential model, the deterministic potential $U(x)$ is represented as follows:

$$U(x) = \frac{b}{2}x^2 + \frac{c}{4}x^4 + \frac{d}{6}x^6 \tag{2}$$

where b, c, and d denote the barrier parameters of a tristable potential function with positive real values.

When a time-delayed feedback item is introduced into Eq. (2), the TFTSR model can be governed by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -bx(t) - cx(t)^3 - dx(t)^5 + ex(t-\tau) + A\cos(2\pi ft) + \sqrt{2D}\xi(t) \tag{3}$$

The time-delayed tristable system potential function can be expressed as

$$U(x) = \frac{b}{2}x^2 + \frac{c}{4}x^4 + \frac{d}{6}x^6 - ex(t-\tau)^2 \tag{4}$$

The delay Fokker-Planck equation is [27].

$$\frac{\partial p(x,t)}{\partial t} = -\frac{\partial\left[h_{\mathrm{eff}}(x)p(x,t)\right]}{\partial x} + \frac{\partial^2[B(x)p(x,t)]}{\partial x^2} \tag{5}$$

where $B(x) = D$ and the conditional average drift $h_{\mathrm{eff}}(x)$ can be expressed as