Journal of Sound and Vibration 420 (2018) 296–314

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Reproducing the nonlinear dynamic behavior of a structured beam with a generalized continuum model

J. Vila [a,b,*], J. Fernández-Sáez [b], R. Zaera [b]

[a] D. Guggenheim School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332, United States
[b] Department of Continuum Mechanics and Structural Analysis, University Carlos III of Madrid, Avda. de la Universidad, 30. 28911 Leganés, Madrid, Spain

ARTICLE INFO

Article history:
Received 3 November 2017
Revised 10 January 2018
Accepted 18 January 2018
Available online XXX

Keywords:
Scale effect
Nonlinear vibration
Finite deformation
Discrete model
Moderate rotations
Nanotechnology

ABSTRACT

In this paper we study the coupled axial-transverse nonlinear vibrations of a kind of one dimensional structured solids by application of the so called Inertia Gradient Nonlinear continuum model. To show the accuracy of this axiomatic model, previously proposed by the authors, its predictions are compared with numeric results from a previously defined finite discrete chain of lumped masses and springs, for several number of particles. A continualization of the discrete model equations based on Taylor series allowed us to set equivalent values of the mechanical properties in both discrete and axiomatic continuum models. Contrary to the classical continuum model, the inertia gradient nonlinear continuum model used herein is able to capture scale effects, which arise for modes in which the wavelength is comparable to the characteristic distance of the structured solid. The main conclusion of the work is that the proposed generalized continuum model captures the scale effects in both linear and nonlinear regimes, reproducing the behavior of the 1D nonlinear discrete model adequately.

1. Introduction

In the last decades, the scientific community has paid attention to micro- and nano-structured materials, such as micro- or nano-electromechanical (MEMS or NEMS) devices [1,2], nanomachines [3–6], as well as in biotechnology and biomedical fields [7–10]. For this reason, there is high technological and scientific interest in the development of a powerful tool for the design of these solids. It is well known that matter is essentially discrete. Therefore, atomistic and molecular dynamic formulations have been used to understand solids behavior. Discrete models and Molecular Dynamics approaches constitute well-known tools to simulate the behavior of microstructured materials and nano-scale elements. However, this calculations are time consuming, even for a relatively low number of particles, if the interaction between them is complex. On the other side, classical continuum models may not capture relevant effects inherent to the microstructure, such as dispersive propagation of waves, size-dependent structural behavior, beaming effect, etc. That is because the classical continuum approach is a scale-free theory.

In contrast, some generalized continuum models have been developed by researchers in this field in order to capture the size effects in the dynamic behavior of microstructured materials. Since the 19th century (works by Cauchy and Voigt), and in the beginning of the 20th century (works by Cosserat brothers) it is possible to find some attempts to capture the effects of microstructure using the continuum equations of elasticity with additional higher-order derivatives. The 1960s supposed a great boost of the topic with the works of Mindlin and Tiersten [11], Kröner [12], Toupin [13,14], or Green

* Corresponding author.
E-mail address: javier.vila@aerospace.gatech.edu (J. Vila).

https://doi.org/10.1016/j.jsv.2018.01.040
0022-460X/

and Rivlin [15]. Several studies have been developed in recent years by making use of strain gradient models, applying its formulation to solve the behavior of nano-shells [16,17], micro and nanoplates [18–21], CNT reinforced elements [22,23], CNTs [24], nano and micro-beams [25–28], micro-pipes [29], among others. Following a different approach, Eringen postulated [30,31] an integral nonlocal constitutive relation for microstructured materials. From this earlier nonlocal theories, he derived a differential version which contains only one additional length scale parameter [32]. This model has been widely used to address different kind of problems related to the mechanical behavior of nanostructures [33–35]. However, several authors have pointed out some inconsistencies arising from the Eringen differential model, both in the static [36–41] and the dynamic regime [42] of nonlocal beams and rods. Recently, Romano et al. [43] showed that, in the majority of cases, the integral formulation of the fully nonlocal elasticity theory of Eringen leads to problems that have to be considered as ill-posed.

All the above works assumed a linear relation between strains and displacements (infinitesimal deformations framework). However, there exist nonlinear solids with an inherent microstructure such as live tissues, highly deformable rubber lattices and other organic compounds that exhibit significant nonlinearities under elastic deformations. In addition, nanomaterials and nanostructures (graphene, CNTs, etc.) can be manufactured with a very low amount of defects, which prevents their collapse under high loads, allowing high deformations and leading to remarkable nonlinear behavior. Nonlinear microstructured materials have a large amount of potential applications in tunable devices [44,45], since the properties of these microstructured solids may be controlled by the amplitude of the phonons propagating through them. Therefore, they could be used for manufacturing non-conventional filters and wave selectors. Nevertheless, only a few attempts have been done up to date to study large strains and rotations (see Dai et al. [46], Karparvarfard et al. [47], Gholipour et al. [48], Andrianov et al. [49], Pal et al. [50] and Dell'Isola et al. [51]). In this respect, Reddy considered [52] the nonlinear von Kármán strains in the analysis of nonlocal formulation of bending of beams and plates under the assumptions of small strains and moderate rotations. Subsequently, this theory has been applied to study the large amplitude free vibration of nanobeams by Şimşek [53,54]. In these works a general formulation of the Eringen nonlocal theory of elasticity valid for finite deformations is not given. However, the Eringen model present the inconsistencies and limitations mentioned above.

In this paper we study the axial-transverse coupled nonlinear vibrations of a kind of one dimensional structured solids, defined as a discrete chain of masses interacting through linear springs, in which size effects play a major role. The paper is organized as follow: in Sec. 2 the discrete model is formulated. The continualization of the governing equations of this discrete model using Taylor series expansion method is presented, showing that it is possible to recover the classical continuous von Kármán beam equations. In Sec. 4, the Inertia Gradient Nonlinear generalized continuum model (IGN), previously postulated by the authors [55], is applied to the analysis of the axial-transverse vibrations of the von Kármán beams (i.e. considering small displacements and moderate rotations). A comparison between results of the discrete model and the predictions of the both generalized continuum one, as well as the classical nonlinear beam model, is given in Sec. 6. We want to remark that this comparison between solutions of a proposed generalized continuum model and a discrete one is not common in the literature. Finally, the main conclusions of the work are presented in Sec. 7.

2. Formulation of a nonlinear discrete beam

In order to study the dynamic behavior of a nonlinear microstructured one dimensional discrete system, a 1-D lattice model is formulated. It consists of P particles equally spaced at distance d with two degrees of freedom in the in-plane motion. Each particle has the same mass m. Particles are joined to first neighbors by longitudinal linear elastic springs, which in turn are coupled by linear elastic torsional springs. A sketch of the chain is shown in Fig. 1. Horizontal and vertical displacements of the *nth* particle from the free equilibrium position are named u_n and v_n, and θ_n is the angle of the *nth* longitudinal spring with its original position.

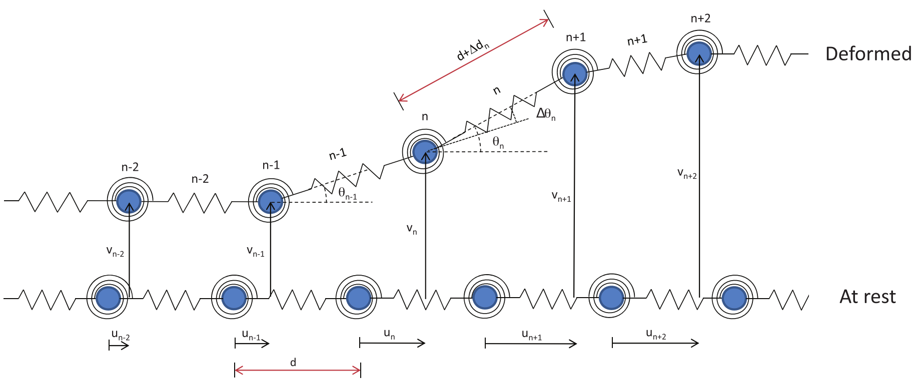

Fig. 1. Sketch of discrete model.