Journal of Sound and Vibration 418 (2018) 144–162

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Simulation of multivariate stationary stochastic processes using dimension-reduction representation methods

Zhangjun Liu [a], Zenghui Liu [b], Yongbo Peng [c,*]

[a] Hubei Key Laboratory of Disaster Prevention and Reduction, China Three Gorges University, Yichang 443002, PR China
[b] College of Civil Engineering & Architecture, China Three Gorges University, Yichang 443002, PR China
[c] State Key Laboratory of Disaster Reduction in Civil Engineering & Shanghai Institute of Disaster Prevention and Relief, Tongji University, Shanghai 200092, PR China

ARTICLE INFO

Article history:
Received 5 October 2017
Received in revised form 9 December 2017
Accepted 15 December 2017

Keywords:
Multivariate stationary stochastic processes
Spectral representation
Proper orthogonal decomposition
Random function
Dimension reduction
Fast Fourier Transform
Wind velocity field

ABSTRACT

In view of the Fourier-Stieltjes integral formula of multivariate stationary stochastic processes, a unified formulation accommodating spectral representation method (SRM) and proper orthogonal decomposition (POD) is deduced. By introducing random functions as constraints correlating the orthogonal random variables involved in the unified formulation, the dimension-reduction spectral representation method (DR-SRM) and the dimension-reduction proper orthogonal decomposition (DR-POD) are addressed. The proposed schemes are capable of representing the multivariate stationary stochastic process with a few elementary random variables, bypassing the challenges of high-dimensional random variables inherent in the conventional Monte Carlo methods. In order to accelerate the numerical simulation, the technique of Fast Fourier Transform (FFT) is integrated with the proposed schemes. For illustrative purposes, the simulation of horizontal wind velocity field along the deck of a large-span bridge is proceeded using the proposed methods containing 2 and 3 elementary random variables. Numerical simulation reveals the usefulness of the dimension-reduction representation methods.

1. Introduction

The stochastic excitations, such as ground motions, turbulence wind velocity and wave fluctuations, acting on complex engineering structures arise to typical dynamic characteristics along the time and the space. They are usually represented by temporal-spatial random fields [1]. Simulation of random fields, therefore, serves as a critical step in stochastic dynamic analysis and reliability assessment of complex engineering structures such as high-rise buildings and long-span bridges. Currently, the random simulation methods referring the Monte Carlo technique receive a sufficient attention in the practices, of which the spectral representation method (SRM) and proper orthogonal decomposition (POD) are both the widely-applied schemes.

The spectral representation method, also known as the harmonic wave superposition method, originated from Rice's work to mathematically formulate a one-dimensional and single-variable random noise [2,3]. Afterwards, the concept of spectral representation was firstly defined by Shinozuka [4]. In case of the simulation of stationary temporal-spatial random field, the

* Corresponding author. 1239 Siping Road, Shanghai 200092, PR China.
E-mail addresses: liuzhangjun@ctgu.edu.cn (Z. Liu), liuzenghuictgu@163.com (Z. Liu), pengyongbo@tongji.edu.cn (Y. Peng).

https://doi.org/10.1016/j.jsv.2017.12.029
0022-460X/

multi-dimensional and multivariate random field is usually dispersed into a one-dimensional and multivariable (1D-nV) stationary random process [5,6]. By the Cholesky decomposition of power spectral density (PSD) matrix, the multivariate stationary random process is represented as the superposition of harmonic waves. On the other hand, an analogous simulation method, named as the proper orthogonal decomposition, was proposed by Di Paola and his colleagues to simulate multivariable stationary stochastic processes [7–9]. Similar to the SRM, POD represents the multivariate stationary stochastic process as the summation of a series of spectral modals on the basis of eigen-decomposition of the PSD matrix. Since the two methods both rely upon the decomposition of PSD matrix implementing the simulation of multivariate stationary random processes, Li and Kareem summarized them into a unified formulation [10].

Both of the SRM and POD can be implemented by two formulas, i.e., the random-phase-angles-based formulas and the orthogonal-random-variables-based formulas, of which the former is more widely-used for simulating stochastic processes and fields, such as ground motion fields [11,12] and turbulence wind fields [7,8,13,14]. One of significant factors is that the random-phase-angles-based formulas can take the advantage of the FFT algorithm, which was firstly proposed by Yang for simulating the random envelope process [15]. Besides, the number of random variables involved in the random-phase-angles-based formulas is half of these in the orthogonal-random-variables-based formulas [16]. Owing to the two factors, the random-phase-angles-based formulas possess a higher efficiency than the orthogonal-random-variables-based formulas. In fact, these two expressions for implementing the SRM and POD can be unified by the spectral theory since both of them can be derived from the Fourier-Stieltjes integral. A detailed discussion on this unified formulation will be provided in the present paper.

Although the SRM and the POD have advantages such as logical theory and simplified algorithms, they are still faced with difficulties when being used to carry out the probability density evolution analysis and reliability assessment of complex structures [17]. There are two main causes hindering them from practical applications. One is the large number of random samplings required to generate the time series of stochastic excitations. The other is the incomplete expansion of stochastic excitations due to the truncation concerning the computational effort, resulting in an insufficient quantification of probability propagation from the stochastic excitations to the dynamic responses of structures [18]. Theoretically, the efficiency of Monte Carlo simulation approaches is independent to the random variables characterizing the dimensions. However, almost all of the pseudo-random numbers generation methods confront the challenges of high-dimension random variables [19].

The dimension-reduction representation of stochastic processes or random fields provides a new manner for overcoming the above-mentioned challenges. Chen and his colleagues developed the formulation of stochastic harmonic functions from the perspective of the second family of spectral representation [20,21]. Frequency components in the harmonic function were defined as random variables whereby just several terms of suppositions could gain an accurate solution with the objective power spectrum. It was proved that the formulation of stochastic harmonic functions had a significant advantage in representing the statistical ensemble of stochastic processes, and applied to the stationary and non-stationary processes [20,22]. Liu et al. implemented the dimension reduction of the random variables both in spectral representation method and orthogonal decomposition (Karhunen-Loeve expansion) through introducing the trigonometric and orthogonal functions as the random constraints [23,24]. The intervention of random functions significantly reduces the random degree inherent in the original spectral representation and orthogonal decomposition, where merely one or two elementary random variables are required.

The previous works upon the dimension-reduction representation just involved in the univariate random process since one-dimensional seismic ground motions at local site were included. While for the random field, one has to deal with the simulation of multivariate random processes. In this paper, the extension of the dimension-reduction representation to simulate multivariate stationary random processes, both using the SRM and POD are proceeded. In order to enhance the efficiency of stochastic suppositions, the FFT algorithm is introduced as well. The remaining sections in this paper are arranged as follows. Section 2 deduces the unified formulation of the SRM and POD from the Fourier-Stieltjes integral of multivariate stationary stochastic processes. The essential relationships between the orthogonal-random-variables-based representation formulas and the random-phase-angles-based simulation formulas are clarified as well. Dimension reduction of the unified formulation and the strategy of selecting representative point sets for elementary random variables are proceeded in Section 3. The procedure of applying the FFT algorithm in the proposed methods to accelerate the numerical simulation is also included in this section. For illustrative purposes, the simulation of horizontal wind velocity field, using the Kaimal wind spectrum and the Davenport coherence function model, along the deck of a large-span bridge is carried out in Section 4. Some concluding remarks are included in Section 5.

2. Spectral decomposition of multivariate stationary stochastic processes

Suppose that $\mathbf{V}(t) = [V_1(t), V_2(t), \cdots, V_n(t)]^{\mathrm{T}}$ is a nV-1D stationary stochastic process with zero mean. The two-sided PSD matrix of $\mathbf{V}(t)$ is defined as follows

$$\mathbf{S_V}(\omega) = \begin{pmatrix} S_{11}(\omega) & S_{12}(\omega) & \cdots & S_{1n}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) & \cdots & S_{2n}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1}(\omega) & S_{n2}(\omega) & \cdots & S_{nn}(\omega) \end{pmatrix} \tag{1}$$