Journal of Sound and Vibration 413 (2017) 291–307

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Validation of vibro-impact force models by numerical simulation, perturbation methods and experiments

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ARTICLE INFO

Article history:
Received 19 May 2017
Revised 7 September 2017
Accepted 15 October 2017
Available online XXX

Keywords:
Vibro-impact dynamics
Impact force models
Kelvin-Voigt impact model
Power-law impact model
Vibro-impacting beam
Lumped mass beam
Experimental validation
Control-based continuation

ABSTRACT

The frequency response of a single degree of freedom vibro-impact oscillator is analyzed using Harmonic Linearization, Averaging and Numeric Simulation, considering three different impact force models: one given by a piecewise-linear function (Kelvin-Voigt model), another by a high-order power function, and a third one combining the advantages of the other two. Experimental validation is carried out using control-based continuation to obtain the experimental frequency response, including its unstable branch.

1. Introduction

Generally speaking, the modeling of vibro-impact systems considers two distinct situations: with and without contact. Hence, one can solve each case separately, connecting them using the contact condition [1]. In Refs. [2,3], the impact condition of piecewise linear systems is used to obtain discrete maps, enabling investigations of bifurcations and stability of periodic motions. However, while it is straightforward to attach solutions of linear systems, the same cannot be said when additional nonlinearities are also present between impacts.

The contact between two bodies can be modeled as a temporary association of the stiffness and damping properties of the colliding bodies, each one modeled as a linear system. Despite its simplicity, this approach has some limitations, mainly the non-zero values of the impact force on the initial and final parts of the contact phase due to dissipative forces. This motivated the development of other contact models such as the one suggested by Hunt and Crossley [4], who derived the damping coefficient as a power function of the impact deformation. This solved the physical inaccuracy of the linear model and lead to an ongoing discussion about the appropriate form of the nonlinear dissipative term [5,6]. Most of the expressions for the dissipative term combine the velocity immediately before impact and the coefficient of restitution with a rational power of the contact deformation, leading to complicated expressions whose analysis is possible only through numerical simulation.

One of the oldest techniques to model impacting systems is to use a coefficient of restitution (CoR) to relate the velocities before and after impact. This classic approach has been used in many applications [7–11]. For instance, Bishop et al. [7] used a

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https://doi.org/10.1016/j.jsv.2017.10.028
0022-460X/

single degree of freedom (SDOF) oscillator with a CoR rule to reproduce the experimental behavior of an impacting cantilever beam around its first natural frequency. Also, the coefficient of restitution, together with a power-law elastic force, can be used to obtain various nonlinear contact force models, such as the one mentioned previously by Hunt and Crossley [4]. Despite its popularity, it should be pointed out that the CoR is not an intrinsic property of the material, depending on the impact velocity [12]. Also, using a simple CoR kinematic rule does not give any direct information about contact forces or stresses.

From the numerical point of view, one can replace discontinuities by smooth equivalent functions and use standard techniques to solve the smoothed model as done by Savi et al. in Ref. [13], where the impact condition is smoothed, or by Elmegård et al. [14], who applied numeric continuation to a smoothed model of a lumped-mass impacting beam. In these cases, while it is safer to use standard ODE solvers provided by accredited sources, it is also necessary to properly tune smooth approximations of discontinuous functions; see Ref. [15] for a discussion on the effect of smoothing functions on the frequency response of oscillators with clearance. There are also several numerical integration algorithms designed for non-smooth systems [16–18], but their comparison is out of the scope of this work.

Vibro-impact oscillators can also be analyzed using common perturbation techniques, such as Harmonic Linearization [19,20], Averaging [21–23], and the Lindstedt-Poincaré method [24]. Besides their widespread use with nonlinear problems, these methods assume weak and smooth nonlinearities, which are not reasonable assumptions for general impacting systems. This limitation motivated the development of non-smooth transformations with respect to state [25,26] and time [27] variables. By these, one can remove non-smooth terms from a certain model, or make them small, allowing the usage of common perturbation methods. An example of such combination can be found in Ref. [8] where non-smooth transformations were used to weaken a near-elastic kinematic impact condition, enabling subsequent application of extended averaging.

In some cases, the applicability of the theoretical/numerical tools mentioned above is accompanied by physical experiments. In Ref. [28] the experimental chaotic behavior of a base-excited cantilever beam with one-sided stop is qualitatively compared with results from numeric simulations of a piecewise linear oscillator. In Ref. [7] the experimental frequency response of a forced cantilever beam with a unilateral constraint is compared with the one obtained by numeric simulation of a SDOF oscillator with a coefficient of restitution, showing reasonable accuracy. The Averaging method has been extensively used to obtain analytical frequency-amplitude expressions for piecewise linear oscillators with one [21,22] and two [23] degrees of freedom.

In Ref. [13] the nonlinear dynamics of a mass-spring system with discontinuous stiffness and damping was analyzed experimentally and numerically by smoothing the impact condition. A similar numerical-experimental analysis was performed by Aguiar and Weber [29], focusing on the behavior of the impact force. Using control-based continuation Bureau et al. [30] obtained experimental frequency responses of a cantilever beam with lumped mass and bilateral constraints. A single-DOF numerical model for this system was proposed by Elmegård et al. [14], who used the experimental data from Ref. [30] to validate the model and predict the existence of an isola, which was later identified experimentally by Bureau et al. [31].

From the overview presented above, one can realize that besides the match between results from particular numerical/analytical techniques with experimental findings there are little efforts on comparing the different paradigms used to model and analyze vibro-impact systems.

The objective and main originality of the present work is to compare different impact force models using analytical, numerical and experimental techniques. The frequency response of a SDOF vibro-impact oscillator is analyzed using Harmonic Linearization and Averaging, considering three different impact force models: one given by the Kelvin-Voigt model (piecewise-linear function), another using a power-law function and a third one combining the strengths of the first two. Experimentally, control-based continuation [30,31] is used to obtain frequency responses of an impacting beam, including its unstable branch. Numerical simulations are used to validate the simple analytic approximations obtained by perturbation methods.

Despite the ability of the mentioned models to produce different qualitative behaviors such as quasiperiodicity and chaos, the analysis presented here is restricted to single-periodic oscillations only.

As the main contribution of this manuscript is to compare different impact force formulations, only the most common models are considered. That is the case of the Kelvin-Voigt model, which is widely used despite its inaccuracies [2,5,23]. The Power-law model can be viewed as a generic version of the compliant force models first derived by Hunt and Crossley [4] and further developed by others [5,32].

2. Experimental setup and procedure

The experimental setup has been described previously in Refs. [30,31] and is shown in Fig. 1. In Fig. 1a and b an electrodynamic shaker (1) [B&K® 4808] is used to apply a harmonic excitation to a platform (2), containing a cantilever beam and a pair of symmetrically located stops (5) to restrain the lateral movement of the beam. The shaker is driven by a power amplifier [B&K® 2712] and is connected to the platform by a stinger. The impacting beam can be seen in detail in Fig. 1c and d, where two DC holding electromagnetic actuators (6) [Magnet-Schultz® G MH X 030] are placed on each side of the lumped mass (4) to execute control-based continuation. The displacement of both platform and lumped mass are measured by two laser sensors (3) [OMRON® ZX-LD40]. Due to the electromagnetic actuators around the lumped mass, its displacement is measured below the mass location, Fig. 1d. A dSPACE® DS1104 R&D controller board is used to perform data acquisition and control-based continuation of the experimental setup.

The experimental frequency response of the impacting beam, shown in Fig. 2a, was obtained by the authors, who repeated some of the experiments done by Refs. [30,31] using control-based continuation. In this model-free approach, the equilibrium states are found by a predictor-corrector algorithm. A non-invasive proportional-derivative controller is used to stabilize the