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Multi-objective parameter identification of Euler–Bernoulli beams under axial load

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ABSTRACT

Identification of physical parameters of the partial differential equation describing transverse vibrations of an axially loaded Euler–Bernoulli beam (EBB) is proposed via a multi-objective optimization formulation and solved by a genetic algorithm. Conflicting objectives such as performance and stability are specifically formulated and optimized simultaneously. Stability is quantified in terms of the solution's time growth factor. Physical parameter sets in the resulting Pareto front approximation represent best trade-offs with respect to the multiple objectives. To compute output error performance objectives, the EBB equation is discretized via finite differences in space and time and reformulated to a state space system. Identifiability is verified by checking regularity of the so-called Fisher information matrix. The identification methodology is capable of determining material parameters, including damping, as well as the axial load from few, spatially concentrated measurements. Its features are demonstrated and successfully validated on specific simulation data and measurement data obtained from a laboratory testbed.

1. Introduction

Many engineering disciplines rely on model-based strategies for system design, simulation, or control. Therefore, models of optimal quality need to be obtained with limited modeling effort. Generic conflicting objectives for the modeling process are high model quality for the system quantities in the relevant domain, consistency of properties such as system stability, low modeling effort, low model complexity, easy parametrization, parameter interpretability, and flexibility with respect to system parameter changes. The modeling process is particularly challenging for distributed-parameter systems, that is, systems whose dynamics are governed by partial differential equations (PDEs).

The main contributions of this paper are (i) the development of a multi-objective optimization methodology to identify physical parameters of an Euler–Bernoulli beam (EBB) model of transverse beam vibrations under constant (but unknown) axial load and known traversal excitation, (ii) the development of suitable performance and stabilizing cost functions including a stability criterion for the physical parameters, and (iii) the formulation of a necessary identifiability condition with respect to the performance objectives. To solve the multi-objective parameter optimization problem, a multi-objective

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genetic algorithm is employed. After the optimization the best possible trade-offs of the represent physical parameter sets are available. The approach aims to keep the necessary measurement effort as small as possible while obtaining high model quality. It is successfully validated at both specific simulation data and measurement data from a laboratory testbed.

Identification of parameters of beam equations has been investigated in various ways. Given the displacement field of a vibrating beam, the authors of Ref. [1] study the identification of the forcing location (forcing function) by the experimental, localized force analysis technique (FAT). Rigid-body parameters (mass, center-of-gravity, inertia tensor) of a softly suspended flexible structure have been identified based on measured vibration data by a modal analysis approach in Ref. [2]. Various techniques to identify viscous damping parameters in linear dynamic models and to assess these methods in an experimental setting in terms of frequency-domain and spatial fit are investigated in Ref. [3]. In Ref. [4], solutions for flexural vibrations in cylindrical rods, both for slender and thick geometries and also considering axial forces are given numerically and are verified by experiments. Refs. [5–7] propose identification approaches to determine the axial loads and boundary condition parameters of Euler–Bernoulli beams from sufficiently many vibration measurements. The required number of spatially distinct measurement positions depends on the number of unknowns in the boundary conditions. In Ref. [8] the authors identify the axial force and stiffness of an Euler–Bernoulli beam under compressive axial load from measured displacement data based on a modal decomposition via a regularized least-squares approach. In Ref. [9] the mechanical properties of a laminated beam are identified using a Ritz–Galerkin approach and the inverse problem is solved by a multi-start global search algorithm. In Ref. [10], a modal inverse method is proposed to identify material parameters of sandwich composite beams considered as Timoshenko beams. The work [11] surveys vibration-based damage identification methods which comprise a special case of the parameter identification problem in elastic structures. More recently, beam structure damage identification has been treated for example in Ref. [12] which focuses on the change of wave propagation properties when damage is present.

In the field of structural model identification and damage detection, multi-objective optimization formulations are used and solved by evolutionary algorithms like genetic algorithms [13–16]. All of these references use Finite Element models. In [13] a multi-objective framework is proposed to identify multiple modal properties (modal frequencies and shapes) simultaneously using the Strength Pareto Evolutionary Algorithm [17]. To quantify and localize the structural damage of beams, the authors of [14] use the modal flexibility [18] as first objective and the quotient between a mode shape change and a frequency change (which only depends on the location of the damage [19]) as a second objective, respectively. The resulting multi-objective optimization problem is solved by the Niched Pareto Genetic Algorithm [20]. A special parameter selection method based on the parameter sensitivity was introduced in [21] in addition to a Finite Element model updating both integrated in a multi-objective optimization framework.

In contrast to the aforementioned works, this contribution aims at obtaining all physical problem parameters (stiffness, damping, axial force, density) of an EBB simultaneously from one or more measurement signals plus the known excitation signal, the problem geometry, and the boundary conditions. Performance and stability of the resulting system discretization are explicitly considered in the multi-objective optimization problem formulation.

The multi-objective identification methodology in this paper is developed in Section 2. First, the EBB equation under axial load with initial and boundary conditions is defined. To perform the multi-objective optimization a simulation model of the EBB equation is needed. The simulation model should be flexible with respect to sensor and actor configurations and effective with respect to simple implementation and numerical accuracy in the context of noisy measurement data. An effective choice for the numerical approximation of the EBB equation is the finite difference (FD) method because its implementation is simple yet flexible, and the accuracy of the second-order approximations is sufficient when applying FD to approximate the temporal and spatial partial derivatives, a system of (implicit) algebraic equations is obtained. To increase flexibility with respect to variations of sensor and actor placement these equations are reformulated into a discrete state space representation.

High model quality can be obtained by minimizing the model output error (the difference between the time signal of the model output and the measured output) for an output error configuration. Identifiability of the parameters is verified by checking regularity of the so-called Fisher information matrix [22,23]. A necessary condition of identifiability is stated. In the next step, the multi-objective parameter identification problem is defined and performance objectives are formulated to minimize the model output error. If only performance objectives are utilized, the identified parameter set may lead to unstable system dynamics even if the model output error is low and the original process is stable. This justifies the formulation of a stabilizing objective (instead of a stability constraint), illustrating the trade-off between performance and stability margin. A stability criterion is defined by utilizing a discrete fundamental solution to the unbounded discretized PDE and by obtaining the so-called dispersion relation [24]. This dispersion relation is a complex polynomial in the temporal growth factor, and its coefficients are functions of the physical parameters. The roots of this polynomial all have to reside inside the unit circle to ensure stability. To formulate bounds for the physical parameters that ensure stability, the Bistritz criterion [25] is used. The stabilizing objective proposed in this paper ensures stability and is also a quantitative measure of stability of the identified parameter set. The multi-objective optimization problem is solved by means of a multi-objective genetic algorithm which produces an approximation of the Pareto front. This enables the engineer to select efficient physical parameter sets without re-optimization and eases understanding of the multi-objective problem.

In Section 3 the developed identification methodology is demonstrated by means of simulation data generated with an undamped beam model as well as actual measurement data from a laboratory testbed. The simulation data example demonstrates the pairing of a performance and a stability objective, whereas the measurement data test case considers two