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Numerical pressure retrieval from velocity measurement of a turbulent tornado-like vortex

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ABSTRACT

This paper presents the computational reconstruction of a three-dimensional distribution of time-averaged pressure of a large-scale tornado-like vortex based on measurement data of time-series velocities. The large-scale tornado simulation facility of Texas Tech University, known as VorTECH, was used to acquire the time-series velocities over a cross-section through the apparatus center as well as the time-series pressure on the floor. The three-dimensional pressure was numerically retrieved using a solution of the pressure Poisson equation discretized by finite element approximation while taking into consideration the effect of turbulence on the pressure in terms of the Reynolds stress. The computed pressure distribution near the floor was validated using the measured floor pressure. The effect of turbulence was found to be significant when attempting to accurately compute pressures using turbulent flow data. Furthermore, it was shown that the computer code is capable of assimilating a measured flow field into the divergence-free field by determining the stationary solution of a functional using the Lagrange multiplier. It was found that the effect of assimilation on the retrieved pressure is not significant.

1. Introduction

A tornado could have a significant impact on buildings and structures not only owing to its wind load but also its pressure difference load. A simple vortex model such as the Rankine vortex model has often been used to conservatively estimate the maximum pressure deficit in structural design; however, real tornadoes have a three-dimensional pressure distribution. Actual pressure distributions of tornadoes have only been obtained at locations near the ground, most of which were coincidentally recorded at weather stations or barometers over which or near which tornadoes had happened to pass. Such records are listed in Table 16.3 of Davies-Jones and Kessler (1974). Moreover, Karstens et al. (2010) successfully determined nine near-ground pressure profiles of tornadoes from 2002 to 2008 using Hardened In-Situ Tornado Pressure Recorder probes. Hoecker (1961) derived a three-dimensional axisymmetric pressure distribution of the Dallas tornado at a distance from the ground in 1957 using a tangential speed distribution from still frames in videos and the cyclostrophic wind equation (balance of radial pressure gradient and centrifugal force). With the advent of particle image velocimetry (PIV) and modern computer technologies, more sophisticated reconstructions of an instantaneous pressure field have been rigorously attempted (van Oudheusden, 2013; van Gent et al., 2017). Recently, the PIV technique has been extensively employed in measuring flow field of a tornado-like vortex. For example, Hashemi Tari et al. (2010) successfully acquired characteristic quantities of turbulence of a rather small-scale tornado-like vortex. Application of PIV to a large-scale tornado-like vortex has also been accomplished using a 1/11 model of a new facility known as WindEEE (Refan and Hangan, 2016). However, the PIV-based measurement is usually limited to rather small-scale flow fields, which impede application to large-scale experiments and actual tornadoes. The aforementioned issues have prevented us from deriving the three-dimensional pressure distribution of large-scale tornado-like vortices, and, consequently, that of actual tornadoes as well.

In this study, the authors have presented a three-dimensional distribution of the time-averaged pressure of a large-scale tornado-like vortex. The pressure has been computationally derived from time-series velocity data measured in the vertical cross-section of a quasi-steady vortex generated in the VorTECH facility of Texas Tech University, USA. The

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computer code employed in this study was developed by Central Research Institute of Electric Power Industry (CRIEPI), Japan (Eguchi et al., 2014). The computation is based on the pressure Poisson equation, which can be derived from the continuity and momentum equations, as has often been done in previous studies (Gresho and Sani, 1987; Sani et al., 2006; van Oudheusden, 2013). The code can also be used to correct a measured flow field such that it strictly satisfies the divergence-free constraint or mass conservation law for incompressible fluids. The code is formulated by only implementing the equations in a three-dimensional Cartesian co-ordinate system, whereas the axisymmetric cylindrical co-ordinate system is ignored. The unique feature of the present study is that it takes the effect of turbulence on the pressure into consideration in the pressure retrieval. The reason is because the turbulence could have an impact on the pressure, as demonstrated in the static-pressure measurements performed by Miller and Comings (1957) in the case of the free turbulent jet.

In the following section 2, we present the theoretical background regarding an assimilation method, with which measured velocities can be projected onto a divergence-free velocity field. Furthermore, it is explained how the time-averaged pressure can be computed using both the time-averaged velocity field and turbulence quantities, *i.e.*, the Reynolds stress. In section 3, we describe the experimental facility as well as the measurement methods used for obtaining the time-series velocity and floor pressure in a large-scale tornado-like vortex. In section 4, we present the computational results of the projected velocity field and computed pressure field. In section 5, we discuss the effect of turbulence on the pressure while comparing the computed near-floor pressure with the measured floor pressure. The final section presents the conclusion.

2. Theoretical background

2.1. Wind field assimilation method

When a velocity field $\mathbf{v}_{\exp}$ obtained in a measurement does not satisfy the continuity equation or divergence-free constraint, we can obtain the divergence-free velocity field $\mathbf{v}$ by determining the stationary point of the following functional $F(\mathbf{v},\lambda)$, as pioneered by Sani et al. (1978) and as explained by Gresho and Sani (2000).

$$F(\mathbf{v},\lambda) = \frac{1}{2}\int_{\Omega}\left|\mathbf{v}-\mathbf{v}_{\exp}\right|^2 d\Omega - \int_{\Omega}\lambda\nabla\cdot\mathbf{v}\,d\Omega \tag{1}$$

where λ is a Lagrange multiplier, and Ω is a fluid domain. At the stationary point, the first variation δF should be zero.

$$\begin{aligned}\delta F &= F(\mathbf{v}+\delta\mathbf{v},\lambda+\delta\lambda) - F(\mathbf{v},\lambda)\\ &\approx \int_{\Omega}\left(\mathbf{v}-\mathbf{v}_{\exp}\right)\cdot\delta\mathbf{v}\,d\Omega - \int_{\Omega}\lambda\nabla\cdot\delta\mathbf{v}\,d\Omega - \int_{\Omega}\delta\lambda\nabla\cdot\mathbf{v}\,d\Omega\\ &= \int_{\Omega}\left(\mathbf{v}-\mathbf{v}_{\exp}+\nabla\lambda\right)\cdot\delta\mathbf{v}\,d\Omega - \int_{\Gamma}\lambda\delta\mathbf{v}\,d\Gamma - \int_{\Omega}\delta\lambda\nabla\cdot\mathbf{v}\,d\Omega = 0\end{aligned} \tag{2}$$

where Γ is the entire surface of a fluid domain. The above equation and the arbitrariness of the variations $\delta\mathbf{v}$ and $\delta\lambda$ result in the following Euler–Lagrange equations.

$$\mathbf{v}-\mathbf{v}_{\exp}+\nabla\lambda = 0 \quad \text{in } \Omega \tag{3}$$

$$\nabla\cdot\mathbf{v} = 0 \quad \text{in } \Omega \tag{4}$$

$$\lambda = 0 \quad \text{on } \Gamma \tag{5}$$

To numerically compute the divergence-free velocity field $\mathbf{v}$, we employ a three-dimensional finite element approximation with eight-node hexahedron isoparametric elements. The velocity and the Lagrange multiplier are respectively interpolated using tri-linear functions Φ_α with respect to node α and piecewise constant functions, along with a stabilization technique (Hughes and Franca, 1987; Eguchi, 2003). The conventional Galerkin formulation for Eqs. (3) and (4) allows us to obtain spatially discretized linear systems for the unknown nodal velocity vector $\{\mathbf{V}\}$ and Lagrange multiplier vector $\{\mathbf{\Lambda}\}$ with the known velocity vector $\{\mathbf{V}_{\exp}\}$ as shown below.

$$[\mathbf{M}]\{\mathbf{V}\} - [\mathbf{M}]\left\{\mathbf{V}_{\exp}\right\} - [\mathbf{C}]\{\mathbf{\Lambda}\} = \{0\} \tag{6}$$

$$[\mathbf{C}]^{\mathrm{T}}\{\mathbf{V}\} = \{0\} \tag{7}$$

The boundary condition of Eq. (5) is implicitly embedded in Eq. (6) as a natural boundary condition. The matrices $[\mathbf{M}]$ and $[\mathbf{C}]$ denote the mass matrix and gradient matrix, which are composed of the element matrices $[\mathbf{M}^e]$ and $[\mathbf{C}^e]$, respectively, as follows.

$$[\mathbf{M}^e] = \int_{\Omega^e}\Phi_\alpha\Phi_\beta\,d\Omega \tag{8}$$

$$[\mathbf{C}^e] = \left[\int_{\Omega^e}\frac{\partial\Phi_\alpha}{\partial x}d\Omega \quad \int_{\Omega^e}\frac{\partial\Phi_\alpha}{\partial y}d\Omega \quad \int_{\Omega^e}\frac{\partial\Phi_\alpha}{\partial z}d\Omega\right]^{\mathrm{T}} \tag{9}$$

where the subscripts α and β indicate the local nodal number ranging from 1 to 8, while Ω^e is the domain of element e. On eliminating $\mathbf{V}$ from Eqs. (6) and (7), the following discrete Poisson equation is obtained.

$$[\mathbf{C}]^{\mathrm{T}}[\underline{\mathbf{M}}]^{-1}[\mathbf{C}]\{\mathbf{\Lambda}\} + \kappa[\mathbf{J}]\{\mathbf{\Lambda}\} = -[\mathbf{C}]^{\mathrm{T}}\left\{\mathbf{V}_{\exp}\right\} \tag{10}$$

As shown above, a diagonal lumped mass matrix $[\underline{\mathbf{M}}]$ is used instead of $[\mathbf{M}]$ to reduce the computational effort and memory storage requirement. Furthermore, the second term along with a non-dimensional constant, κ, is specifically used to suppress the numerical instability inherent to the present finite element approximation by using tri-linear functions for the velocity and piecewise constant functions for the Lagrange multiplier. The element vector with respect to the element e, $\kappa([\mathbf{J}]\{\mathbf{\Lambda}\})^e$, can be expressed as:

$$\kappa([\mathbf{J}]\{\mathbf{\Lambda}\})^e = \frac{\kappa}{b^2}\left[\sum_{i=1}^{M^e}\left(\Lambda^{e(0)} - \Lambda^{e(i)}\right)S_{e(i)}^{\frac{3}{2}}\right] \tag{11}$$

where b is an average mesh size, and M^e is the number of interior inter-element faces of the element e, $S_{e(i)}$ is the contact area of the i-th inter-element face of element e, $\Lambda^{e(0)}$ is the Lagrange multiplier of element e, and $\Lambda^{e(i)}$ is the Lagrange multiplier of the i-th element contacting element e. The linear system of Eq. (10) is solved using a commercial matrix solver, SAMG, developed by the Fraunhofer Institute, Germany. On substituting the solution $\{\mathbf{\Lambda}\}$ into the following equation, we obtain the divergence-free velocity $\{\mathbf{V}\}$.

$$\{\mathbf{V}\} = \left\{\mathbf{V}_{\exp}\right\} + [\underline{\mathbf{M}}]^{-1}[\mathbf{C}]\{\mathbf{\Lambda}\} \tag{12}$$

2.2. Pressure retrieval method

We assume that the time-series velocities are available throughout the fluid domain *via* measurements. The divergence-free counterpart of the time-averaged velocities can also be obtained using the aforementioned method. The pressure retrieval method employed is based on the unsteady three-dimensional incompressible Navier–Stokes equations and the incompressibility constraint below.

$$\frac{\partial\mathbf{u}}{\partial t} = -(\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla p + \nu\nabla^2\mathbf{u} \quad \text{in } \Omega \tag{13}$$

$$\nabla\cdot\frac{\partial\mathbf{u}}{\partial t} = 0 \quad \text{in } \Omega \tag{14}$$

where $\mathbf{u}$ and p respectively denote the velocity and kinematic pressure (pressure divided by fluid density), while t and ν respectively represent the time and kinematic viscosity. The above equations are supplemented with a velocity boundary condition because the velocity values are available over the entire fluid boundary Γ as well as in the entire fluid domain Ω.