Pacific Science Review A: Natural Science and Engineering xxx (2016) 1–9

HOSTED BY

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Contents lists available at ScienceDirect

Pacific Science Review A: Natural Science and Engineering

journal homepage: www.journals.elsevier.com/pacific-science-review-a-natural-science-and-engineering/

Range-valued fuzzy colouring of interval-valued fuzzy graphs

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ARTICLE INFO

Article history:
Received 2 August 2016
Accepted 8 September 2016
Available online xxx

Keywords:
Interval-valued fuzzy graph
Complete interval-valued fuzzy graph
Interval-valued fuzzy neighbourhood
α-Cut graph of interval-valued fuzzy graph
Range-valued fuzzy colouring

ABSTRACT

In this paper, we observed that range-valued fuzzy graphs are more popular than fuzzy graphs following the level of participation of nodes and branches in the range [0, 1] other than at a point in fuzzy graphs. Colouring interval-valued fuzzy graphs has a few general applications. In this paper, another idea of using colouring for interval-valued fuzzy graphs is presented. Additionally, a few vital terms, such as power cut graphs, interval-valued fuzzy colour, and chromatic numbers of interval-valued fuzzy graphs, are depicted. Some important theorems are discussed. This procedure is utilized to colour the India political map, revealing the power of the relationship within the nation. Additionally, another sort of traffic signal system is analysed.

1. Introduction

In our daily life, the colouring of a graph is the most significant component of research in optimization technology and is used for various applications, viz. administrative sciences, wiring printed circuits [11], resource allocation [5], arrangement problems [1,7,10], and so on. These problems are represented by proper crisp graphs and are analysed by colouring these graphs. In the usual graph colouring problem, nodes receive the minimum number of colours such that two adjacent nodes do not have the same colour. A few studies discuss this point [8,9,15,31]. An interval-valued fuzzy graph representation is better than a crisp graph version. Interval-valued fuzzy graphs suitably represent every event.

Interval-valued fuzzy graph theory has a broad number of areas. Interval-valued fuzzy set notations and their properties was introduced by Zadeh in 1975 [3]. Interval-valued fuzzy sets are more advanced than fuzzy sets and more completely eliminate doubt. At present, Akram uses interval valued fuzzy graphs [28].

Bhutan and Rosenfeld have worked on strong arcs in fuzzy graphs [4] and have researched their properties. Cozy has utilized fuzzy graphs in the assessment and advancement of systems [8]. With a different approach, Samanta and Pal introduced fuzzy planar graphs [22]. Additionally, they presented many changes in fuzzy graphs in various studies [17–21,23–27].

In a literature survey, one of the helpful problems, the traffic signal problem, was solved using the crisp graph colouring method. However, compared to other paths, some still deal with the traffic signal problem. Additionally, two paths can occasionally be opened at the same time with a warning. Here “Crowder” and “warning” are fuzzy terms. By colouring fuzzy graphs, Munoz et al. [14] approached the traffic signal problem. In their paper, fuzzy graphs are plotted with crisp nodes and fuzzy branches. Then, α-cut [13] of these fuzzy graphs is coloured by the technique of crisp graph colouring. For distinct values of α, we have dissimilar crisp graphs, and these crisp graphs are coloured. Therefore, the chromatic number changes for similar fuzzy graphs according to distinct values of α. Additionally, Bershtein and Bozhenuk [2] planned a method to colour fuzzy graphs.

In the present paper, a new idea to colour an interval-valued fuzzy graph is presented. Here, “range-valued fuzzy colour” is defined. An interval-valued fuzzy graph is coloured by the range-valued fuzzy colour depending on the power of a branch incident to a node. This latest colouring idea is used to colour the political map and solve the latest type of traffic signal colouring problem. In this paper the ‘power cut graph’ of interval-valued fuzzy graphs is measured. Some enthusiastic theorems on this type of graph are considered. Then, range-valued fuzzy colouring of interval-valued fuzzy graphs is demonstrated.

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Peer review under responsibility of Far Eastern Federal University, Kangnam University, Dalian University of Technology, Kokushikan University.

http://dx.doi.org/10.1016/j.psra.2016.09.006

2. Preliminaries

2.1. Graph

Definition 2.1. A graph G is a triplet that contains a set $V \neq \phi$, a branch set E, and a connection that links each branch by two nodes (not as a particular matter of course) called its end points.

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2.3. Node colouring and chromatic number

Definition 2.2. The node colouring of a graph G is the consignment of labels or colours to each node of a graph such that no branch links two similarly coloured nodes. The general type of node colouring search minimizes the number of colours for a graph. This type of colouring is known as least node colouring, and the lowest number of colours with which the nodes of a graph G can be coloured is called the chromatic number. The chromatic number of a graph G is denoted by $\chi(G)$.

2.4. Fuzzy set

Definition 2.3. Let P be a universal set. Then, the fuzzy set F on P is indicated by function $\eta : P \to [0,1]$, which is called the membership function. A fuzzy set is represented by $F = (P, \eta)$.

2.5. Fuzzy graph

Definition 2.4. A fuzzy graph [29] $\delta = (V, \psi, \rho)$ is a set $V \neq \phi$ together with the functions $\psi : V \to [0,1]$ and $\rho : V \times V \to [0,1]$ such that for all $m, n \in V, \rho(m,n) \leq \min\{\psi(m), \psi(n)\}$, where $\psi(m)$ and $\rho(m,n)$ denote the membership values of the node m and branch (m,n) in δ, respectively.

2.6. Path in a fuzzy graph

Definition 2.5. A path in a fuzzy graph is an arrangement of different nodes $m_0, m_1, \ldots\ldots, m_n$ such that $\rho(m_{i-1}, m_i) > 0, 1 \leq i \leq n$. The fuzzy path is said to be a fuzzy cycle if m_0 and m_n overlap.

The original crisp graph of the fuzzy graph $\delta = (V, \psi, \rho)$ is denoted as $\delta^* = (V, \psi^*, \rho^*)$, where $\psi^* = \{x \in V | \psi(x) > 0\}$ and $\rho^* = \{(x,y) \in V \times V | \rho(x,y) > 0\}$. For the original fuzzy graph, $\psi^* = V$.

2.7. Complete fuzzy graph

Definition 2.6. A fuzzy graph $\delta = (V, \psi, \rho)$ is said to be complete if $\rho(x,y) = \min\{\psi(x), \psi(y)\}$ for all $x, y \in V$, where (x,y) denotes the branch between nodes x andy.

2.8. Fuzzy sub graph

Definition 2.7. The fuzzy graph $\delta_a = (V, \psi_a, \rho_a)$ is called a fuzzy sub graph of $\delta = (V, \psi, \rho)$ if $\psi_a(m) \leq \psi(m)$ for all m and $\psi_a(m,n) \leq \psi(m,n)$ for all branches $(m,n), m, n \in V$.

2.9. Interval-valued fuzzy set

Definition 2.8. Let $V \neq \phi$ and $\psi^- : V \to [0,1]$ and $\psi^+ : V \to [0,1]$ be the mappings such that $\psi^-(v) \leq \psi^+(v)$ for all $v \in V$. The interval-valued fuzzy set on V is denoted as $(V, [\psi^-, \psi^+])$ and is defined as $(V, [\psi^-, \psi^+]) = \{(v, [\psi^-, \psi^+]) | v \in V\}$.

2.10. Interval-valued fuzzy graph

Definition 2.9. Interval-valued fuzzy graph $I_{FG} = (V, [\psi^-, \psi^+], [\rho^-, \rho^+])$ (IVFG) is a set $V \neq \varphi$ together with the functions $\psi^- : V \to [0,1]$, $\psi^+ : V \to [0,1]$, $\rho^- : V \times V \to [0,1]$ and $\rho^+ : V \times V \to [0,1]$ such that for all $x, y \in V$ $\rho^-(x,y) \leq \min\{\psi^-(x), \psi^-(y)\}$ and $\rho^+(x,y) \leq \min\{\psi^+(x), \psi^+(y)\}$ for every branch (x,y).

2.11. Complete interval-valued fuzzy graph

Definition 2.10. The complete interval-valued fuzzy graph of $I_{FG} = (V, [\psi^-, \psi^+], [\rho^-, \rho^+])$ (IVFG) is a set $V \neq \varphi$ together with the functions $\psi^- : V \to [0,1]$, $\psi^+ : V \to [0,1]$, $\rho^- : V \times V \to [0,1]$ and $\rho^+ : V \times V \to [0,1]$ such that for all $x, y \in V \rho^-(x,y) = \min\{\psi^-(x), \psi^-(y)\}$ and $\rho^+(x,y) = \min\{\psi^+(x), \psi^+(y)\}$ for every branch (x,y).

2.12. α-Cut graph of interval-valued fuzzy graph

Definition 2.11. For $0 \leq \alpha \leq 1$, α-cut graph of interval-valued fuzzy graph $I_{FG} = (V, [\psi^-, \psi^+], [\rho^-, \rho^+])$ is a crisp graph $I_{FG_\alpha} = (V_\alpha, E_\alpha)$ such that $V_\alpha = \{x \in V | [\psi^-(x), \psi^+(x)] \geq [\alpha, \alpha]\}$ and $E_\alpha = \{(x,y), | [\rho^-(x,y), \rho^+(x,y)] \geq [\alpha, \alpha]\}$.

2.13. Interval-valued fuzzy neighbourhood

Definition 2.12. Interval-valued fuzzy neighbourhood of a node v of an interval-valued fuzzy graph $I_{FG} = (V, [\psi^-, \psi^+], [\rho^-, \rho^+])$ is an interval-valued fuzzy set $N(v) = (X_v, m_v)$, where $X_v = \{u | [\rho^-(v,u), \rho^+(v,u)] > [0,0]\}$ and $m_v : X_v \to [0,1]$ is defined by $m_v(u) = \mu(v,u)$.

Fuzzy star is defined in [30].

2.14. Interval-valued fuzzy star

Definition 2.13. A power neighbourhood of a node p is a node q such that (p,q) is a power full branch. An interval-valued fuzzy graph I_{FG} is said to be the interval-valued fuzzy star if each node of I_{FG} has precisely one power neighbour in I_{FG}.

3. Power cut graph of interval-valued fuzzy graphs

In this part, the α-power cut graph of I_{FG} is defined with an example.

For an interval-valued fuzzy graph $I_{FG} = (V, [\psi^-, \psi^+], [\rho^-, \rho^+])$, a branch $(m,n), m, n \in V$ is said to be independently powerful if $(0.5)\min\{\psi^+(m), \psi^+(n)\} \leq \rho^-(m,n)$ and $(0.5)\min\{\psi^+(m), \psi^+(n)\} \leq \rho^+(m,n)$. Otherwise, it is independent and powerless. The power of a branch (p,q) in an interval-valued fuzzy graph $I_{FG} = (V, [\psi^-, \psi^+], [\rho^-, \rho^+])$ is denoted by $\tau_{(p,q)}$ and is defined as $\tau_{(p,q)} = [\tau^-_{(p,q)}, \tau^+_{(p,q)}]$, where $\tau^-_{(p,q)} = \frac{\rho^-(p,q)}{\min\{\psi^+(p), \psi^+(q)\}}$ and $\tau^+_{(p,q)} = \frac{\rho^+(p,q)}{\min\{\psi^+(p), \psi^+(q)\}}$. Again the power of a node w is denoted by τ_w and defined as $\tau_w = [\tau^-_w, \tau^+_w]$, where τ^-_w is the maximum value along its membership value $\psi^-(w)$, and the powers $\tau^-_{(w,x)}$ of branches (w,x) incident to w and τ^+_w are the maximum values along its