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Fuzzy risk analysis in familial breast cancer using a similarity measure of interval-valued fuzzy numbers

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A B S T R A C T

In this work, a new similarity measure was proposed based on the heights and areas of interval-valued trapezoidal fuzzy numbers. Some properties corresponding to the proposed similarity measure were illustrated. A comparison with the different existing similarity-measurement techniques demonstrated that the proposed method gave better results, overcoming the drawbacks of the existing methods. The proposed similarity-measurement technique was applied to the prediction of risk in the burning problem of Familial Breast Cancer (FBC). Finally, a numerical illustration for FBC was given using interval-valued trapezoidal fuzzy numbers.

1. Introduction

Risk is one type of uncertainty in which some of the probabilities describe a loss or another undesirable outcome. It is usually determined in a probabilistic way. Currently, a number of researchers are working on fuzzy risk analysis. In 1984, Schmucker [12] proposed fuzzy risk analysis to address the uncertainty involved in a system. After that, Kangaari and Riggs [20] presented a method to construct risk assessment using linguistic terms in 1989. Chen [27] presented a method for subjective mental-workload assessment and fuzzy risk analysis. Recently, it has been recognized that the similarity measure is an important tool in the field of risk analysis; a number of researchers are currently working in this area. In 2003, Chen and Chen [22] presented a fuzzy risk-analysis method based on similarity measures of generalized fuzzy numbers. Tang and Chi [29] presented a method for predicting multilateral-trade credit risks. Wang and Elhag [32] presented a fuzzy TOPSIS method based on alpha-level sets with an application for bridge risk assessment. Patra and Mondal [13] presented a method to conduct fuzzy risk analysis using area- and height-based similarity measures on generalized trapezoidal fuzzy numbers. Then, Patra and Mondal [14] presented a method to conduct risk analysis in diabetes prediction based on a new approach involving the ranking of generalized trapezoidal fuzzy numbers.

Moreover, in recent years, some researchers have focused on the topic of interval-valued fuzzy numbers. Gorzalczany [16] presented an approximate reasoning method based on interval-valued fuzzy sets. Guijun and Xiaoping [31] introduced some applications of interval-valued fuzzy numbers and interval-distribution numbers. Wang and Li [7] presented a method based on the correlation and information energy of interval-valued fuzzy numbers. Hong and Lee [2] presented algebraic properties and a distance measure for interval-valued fuzzy numbers. Chen [24] presented a method for handling the similarity-measurement problems of interval-valued fuzzy numbers. Chen and Chen [23] presented a method to measure the similarity between interval-valued fuzzy numbers. Chen and Chen [26] also presented a method to measure the similarity between interval-valued trapezoidal fuzzy numbers. Chen and Sanguansat [25] presented a method to measure the similarity between interval-valued trapezoidal fuzzy numbers.

Familial Breast Cancer is a burning problem in the modern era. The causes of breast cancer are not fully known. However, researchers have identified a number of factors that increase (or decrease) the chances of developing breast cancer. These are called risk factors. Breast cancer is complex and likely caused by a combination of risk factors. Thus, the measurement of risk of familial breast cancer is essential and appropriate in the current era.

In this paper, we introduce a new similarity-measurement technique that uses the area and height of the difference between

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two interval-valued trapezoidal fuzzy numbers. After that, some properties of this method are derived. Using fifteen different sets of interval-valued trapezoidal fuzzy numbers, it is shown that the proposed method is better than the existing methods. Finally, the proposed method is used to analyse the fuzzy risk in a real-life problem of Familial Breast Cancer.

The proposed research paper is organized as follows. In Section 2, we introduce the preliminaries on interval-valued fuzzy numbers. In Section 3, the limitations of the existing literature on measuring the similarity between two interval-valued fuzzy numbers are discussed. In Section 4, a new method of similarity measurement is proposed. In Section 5, some properties of the proposed similarity measure between interval-valued fuzzy numbers are discussed. In Section 6, the proposed similarity measure is compared with the existing methods. In Section 7, the proposed similarity measure is applied in a Familial Breast Cancer problem. In Section 8, the paper is concluded.

2. Preliminaries of fuzzy risk on interval-valued fuzzy numbers

Some basic concepts of interval-valued fuzzy numbers are briefly reviewed in this section. Based on Yao and Lin [11], an interval-valued trapezoidal fuzzy number $\tilde{A}$ can be represented by $\tilde{A} = [\tilde{A}^L, \tilde{A}^U] = [(a_1^L, a_2^L, a_3^L, a_4^L, \widehat{w}_{\tilde{A}}^L), (a_1^U, a_2^U, a_3^U, a_4^U, \widehat{w}_{\tilde{A}}^U)]$, where $\tilde{A}^L$ denotes the lower interval-valued trapezoidal fuzzy number, $\tilde{A}^U$ denotes the upper interval-valued trapezoidal fuzzy number and $\tilde{A}^L \subset \tilde{A}^U$. Lower and upper interval-valued trapezoidal fuzzy numbers are generalized trapezoidal fuzzy numbers. A plot of the membership function of the interval-valued trapezoidal fuzzy number $\tilde{A}$ is shown in Fig. 1.

(1) If $\tilde{A}^L = \tilde{A}^U$, then the interval-valued trapezoidal fuzzy number $\tilde{A}$ is a generalized trapezoidal fuzzy number and reduces to $(a_1, a_2, a_3, a_4, \widehat{w}_{\tilde{A}})$.
(2) If $a_1 = a_2 = a_3 = a_4$ and $\widehat{w}_{\tilde{A}} = 1$, then the interval-valued trapezoidal fuzzy number $\tilde{A}$ is a crisp value.
(3) If $a_1 < a_2 = a_3 < a_4$, then the interval-valued trapezoidal fuzzy number $\tilde{A}$ is a triangular interval-valued fuzzy number.

Now, it is assumed that there are two interval-valued trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, i.e., $\tilde{A} = [\tilde{A}^L, \tilde{A}^U] = [(a_1^L, a_2^L, a_3^L, a_4^L, \widehat{w}_{\tilde{A}}^L), (a_1^U, a_2^U, a_3^U, a_4^U, \widehat{w}_{\tilde{A}}^U)]$ and $\tilde{B} = [\tilde{B}^L, \tilde{B}^U] = [(b_1^L, b_2^L, b_3^L, b_4^L, \widehat{w}_{\tilde{B}}^L), (b_1^U, b_2^U, b_3^U, b_4^U, \widehat{w}_{\tilde{B}}^U)]$, where $a_i^L, a_i^U, b_i^L, b_i^U$ are real values between 0 and 1, $(1 \le i \le 4, 0 \le \widehat{w}_{\tilde{A}}^L \le \widehat{w}_{\tilde{A}}^U \le 1$ and $0 \le \widehat{w}_{\tilde{B}}^L \le \widehat{w}_{\tilde{B}}^U \le 1)$. Then, according to Chen and Sanguansat [25], some arithmetic operations between $\tilde{A}$ and $\tilde{B}$ are defined as follows.

Here, the addition operator, subtraction operator, multiplication operator, and division operator, denoted by $\oplus$, $\ominus$, $\otimes$ and $\oslash$, respectively, on interval-valued trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$, are defined as

(i) Interval-valued fuzzy-number addition $\oplus$:

$$\begin{aligned}\tilde{A} \oplus \tilde{B} &= \left[\left(a_1^L, a_2^L, a_3^L, a_4^L, \widehat{w}_{\tilde{A}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U, \widehat{w}_{\tilde{A}}^U\right)\right] \oplus \left[\left(b_1^L, b_2^L, b_3^L, b_4^L, \widehat{w}_{\tilde{B}}^L\right), \left(b_1^U, b_2^U, b_3^U, b_4^U, \widehat{w}_{\tilde{B}}^U\right)\right] \\ &= \left[\left(a_1^L + b_1^L - a_1^L b_1^L, a_2^L + b_2^L - a_2^L b_2^L, a_3^L + b_3^L - a_3^L b_3^L, a_4^L + b_4^L - a_4^L b_4^L; min\left(\widehat{w}_{\tilde{A}}^L, \widehat{w}_{\tilde{B}}^L\right)\right),\right. \\ &\left.\left(a_1^U + b_1^U - a_1^U b_1^U, a_2^U + b_2^U - a_2^U b_2^U, a_3^U + b_3^U - a_3^U b_3^U, a_4^U + b_4^U - a_4^U b_4^U; min\left(\widehat{w}_{\tilde{A}}^U, \widehat{w}_{\tilde{B}}^U\right)\right)\right];\end{aligned}$$

(ii) Interval-valued fuzzy-number subtraction $\ominus$:

$$\begin{aligned}\tilde{A} \ominus \tilde{B} &= \left[\left(a_1^L, a_2^L, a_3^L, a_4^L, \widehat{w}_{\tilde{A}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U, \widehat{w}_{\tilde{A}}^U\right)\right] \ominus \left[\left(b_1^L, b_2^L, b_3^L, b_4^L, \widehat{w}_{\tilde{B}}^L\right), \left(b_1^U, b_2^U, b_3^U, b_4^U, \widehat{w}_{\tilde{B}}^U\right)\right] \\ &= \left[\left(a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L; min\left(\widehat{w}_{\tilde{A}}^L, \widehat{w}_{\tilde{B}}^L\right)\right), \left(a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U; min\left(\widehat{w}_{\tilde{A}}^U, \widehat{w}_{\tilde{B}}^U\right)\right)\right];\end{aligned}$$

(iii) Interval-valued fuzzy-number multiplication $\otimes$:

$$\begin{aligned}\tilde{A} \otimes \tilde{B} &= \left[\left(a_1^L, a_2^L, a_3^L, a_4^L, \widehat{w}_{\tilde{A}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U, \widehat{w}_{\tilde{A}}^U\right)\right] \otimes \left[\left(b_1^L, b_2^L, b_3^L, b_4^L, \widehat{w}_{\tilde{B}}^L\right), \left(b_1^U, b_2^U, b_3^U, b_4^U, \widehat{w}_{\tilde{B}}^U\right)\right] \\ &= \left[\left(a_1^L \times b_1^L, a_2^L \times b_2^L, a_3^L \times b_3^L, a_4^L \times b_4^L; min\left(\widehat{w}_{\tilde{A}}^L, \widehat{w}_{\tilde{B}}^L\right)\right), \left(a_1^U \times b_1^U, a_2^U \times b_2^U, a_3^U \times b_3^U, a_4^U \times b_4^U; min\left(\widehat{w}_{\tilde{A}}^U, \widehat{w}_{\tilde{B}}^U\right)\right)\right];\end{aligned}$$

(iv) Interval-valued fuzzy-number division $\oslash$:

$$\begin{aligned}\tilde{A} \oslash \tilde{B} &= \left[\left(a_1^L, a_2^L, a_3^L, a_4^L, \widehat{w}_{\tilde{A}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U, \widehat{w}_{\tilde{A}}^U\right)\right] \oslash \left[\left(b_1^L, b_2^L, b_3^L, b_4^L, \widehat{w}_{\tilde{B}}^L\right), \left(b_1^U, b_2^U, b_3^U, b_4^U, \widehat{w}_{\tilde{B}}^U\right)\right] \\ &= \left[\left(a_1^L / b_4^L, a_2^L / b_3^L, a_3^L / b_2^L, a_4^L / b_1^L; min\left(\widehat{w}_{\tilde{A}}^L, \widehat{w}_{\tilde{B}}^L\right)\right), \left(a_1^U / b_4^U, a_2^U / b_3^U, a_3^U / b_2^U, a_4^U / b_1^U; min\left(\widehat{w}_{\tilde{A}}^U, \widehat{w}_{\tilde{B}}^U\right)\right)\right],\end{aligned}$$

where $a/b = a/b$ when $a \le b$, and $a/b = 1$ when $a > b$.

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