Pacific Science Review A: Natural Science and Engineering xxx (2016) 1–8

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Contents lists available at ScienceDirect

Pacific Science Review A: Natural Science and Engineering

journal homepage: www.journals.elsevier.com/pacific-science-review-a-natural-science-and-engineering/

A finite element one-dimensional kinematic wave rainfall-runoff model

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ARTICLE INFO

Article history:
Received 10 August 2016
Received in revised form
21 November 2016
Accepted 24 November 2016
Available online xxx

Keywords:
Kinematic wave
Galerkin's residual FEM
US SCS method
Rainfall-runoff model

ABSTRACT

This paper introduces a simplified distributed rainfall-runoff model based on the combination of a finite element model with the US SCS method. Excess rainfall is estimated from rainfall, soil and land-use properties according to the SCS. The approximation of river flow uses finite elements, while overland and channel flows are simulated by one-dimensional kinematic wave equations. The finite element algorithm for solving the one-dimensional kinematic wave equations is based on lumped schemes and a third order Runge–Kutta method. The proposed model is applied to estimate the flood flow in the Tra Khuc River Basin. The obtained results show the promise of this method for practical application.

1. Introduction

The need for tools that can simulate the influence of the spatial distribution of rainfall and natural river basin properties on runoff processes created interest in, and initiated the development of, hydrodynamic rainfall-runoff models [1,6]. One of the basic assumptions for such models regards the presence of a continuous layer of water moving over the entire surface of the catchment. Although observations show that such conditions are rare, the assumption can be relaxed by considering the total flow to result from many small plots draining into a fine network of small channels.

The actual physical flow processes may be quite complicated, but for practical purposes, the simplifications of a full hydrodynamic model can be used. As a common way of getting reasonable results, one-dimensional kinematic wave models [3,5,11] are often selected. These can be solved by different methods.

Due to the flexibility of the finite element in simulation of spatially variable parameters of the watershed, the finite element method (FEM) is often applied for solution of one-dimensional kinematic wave equations [2,6,7,11].

Application of FEM to one-dimensional kinematic wave equations raises several problems related to the stability of the solution (numerical oscillation) due to non-symmetric first spatial derivative terms in the kinematic wave equations when using spatial interpolation functions and a temporal approximation for a system of ordinary differential equations.

One way to improve the stability and accuracy of the method is through the choice of space interpolation functions. Blandford and Meadows [3] introduced quadratic (one element and three nodes) and cubic schemes (one element and four nodes) for finite element simulation of kinematic surface runoff. Jaber and Mohtar [7] used linear, lumped and upwind schemes for spatial approximation and the enhanced explicit scheme for temporal discretization for a one-dimensional kinematic wave solution. The result of this study showed that the lumped scheme significantly improved the solution without any reduction in solution accuracy. The improvement in the lumped scheme result is attributed to the sparse diagonal matrix that eliminates numerical noise from off-diagonal terms. They analysed the stability of the different schemes for equidistant elements through Fourier analysis and numerical experiments and concluded that the lumped scheme of the Galerkin finite element method is most suitable for solution of one-dimensional kinematic wave equations. The results of the study of Jaber and Mohtar [7] are

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Peer review under responsibility of Far Eastern Federal University, Kangnam University, Dalian University of Technology, Kokushikan University.

http://dx.doi.org/10.1016/j.psra.2016.11.001

the scientific basis for selection of spatial interpolation functions for development of the finite element one-dimensional kinematic wave rainfall-runoff model.

Another way to develop the FEM rainfall-runoff model is in the choice of the equations simulating the flow in the channel network [6]. In this research, overland flow is simulated using a kinematic wave model, and the finite-element formulation of variable width and variable slope is used to solve the resulting equations. The flow through the network of channels is simulated by solving the full Saint-Venant equations, using the finite element method. The results of the research show the applicability of the proposed formulation.

The FEM is a general and effective technique for transforming partial differential equations into systems of ordinary differential equations [9]. Application of FEM continues to be an interesting subject for solving different types of hydrodynamic wave equations [8,10].

A mountainous river basin with a steep slope is suitable for kinematic wave simulation. During the typically brief flood interval, the interaction between surface and underground flow can be neglected and the base flow is assumed to be stable. The rainfall-runoff simulation could thus be simply simulated by a coupling method that integrates the SCS method for estimating the excess rainfall and the finite element method for solution of the kinematic wave equations simulating overland and channel flow routing processes. A detailed description of this method is presented in the following sections.

2. A finite element one-dimensional kinematic wave rainfall-runoff model

2.1. Finite element kinematic wave model

2.1.1. The one-dimensional kinematic wave equation for overland runoff and river flow simulation

One-dimensional kinematic wave equations are often used to simulate the rainfall-runoff process in small and average-size river basins with steep slopes. The one-dimensional kinematic wave equations are normally written in the form of a continuity equation:

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=r(x,t) \tag{1}$$

and the equation of motion for overland and river flow is:

$$S_0=S_f \tag{2}$$

Using Manning's equation, unit-width flow (q) and flow depth (h) in Equation (1) are related by the following equation:

$$q=\alpha h^{\beta} \tag{3}$$

where h = flow depth (m); q = unit-width flow for overland and river flow (m^2/s); $r(x,t)$ = excess rainfall rate (for overland flow in m/s) or lateral flow (for river flow in m/s); $\alpha=(S_o^{1/2}/n)$; $\beta=5/3$; n is Manning roughness coefficient ($m^{1/3}$/s); S_o is the surface or bottom slope, which equals the friction slope S_f for the kinematic wave approximation; x = spatial coordinate (m) and t = time (s).

2.1.2. Finite element approximation of one-dimensional kinematic wave equations

The principle of spatial discretization for the one-dimensional kinematic wave model using the FEM method is dividing the river basin into "strips" that represent flow direction on the surface of the water flow [11]. Each strip is then divided into computational elements based on the characteristics (e.g., slope) of the basin so that (the slope of) each element is approximately homogeneous. Dividing the watershed into "strips" based on flow direction allows us simulate overland flow by one-dimensional kinematic wave equations.

For each computational element, the variables h(x,t) and q(x,t) are approximated in the form:

$$h(x,t)\approx\widehat{h}=\sum_{i=1}^{n}N_i(x)h_i(t);\quad q(x,t)\approx\widehat{q}=\sum_{i=1}^{n}N_i(x)q_i(t) \tag{4}$$

where

- $N_i(x)$ = functions defined on a spatial interval (element).
- n = number of nodes of spatial functions (for linear and lumped schemes, (n = 2)).

Galerkins residual FEM is based on the principle that the solution residuals should be orthogonal to a set of weighting functions:

$$\int_{\Omega}\sum_{i=1}^{n}\left\{\frac{dh_i}{dt}N_i+q_i\frac{\partial N_i}{\partial x}-r_i\right\}N_i dx=0 \tag{5}$$

The approximation equation (Eq. (4)) combined with the integral equation (Eq. (5)) transforms the partial differential equation (Eq. (1)) into a system of ordinary differential equations, which for each element (Eq. (5)) takes the form:

$$\mathbf{A}^{(e)}\frac{d\mathbf{h}}{dt}+\mathbf{B}^{(e)}\mathbf{q}-\mathbf{f}^{(e)}=0 \tag{6}$$

For the linear scheme, the spatial interpolation functions can be defined as $N_1(x)=1-y$ and $N_2(x)=y$, where $y=x/l$; l is the length of the element.

In this case, the matrixes of Eq. (6) can be written as:

$$\mathbf{B}^{(e)}=\frac{1}{2}\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix};\quad \mathbf{A}^{(e)}=\begin{bmatrix}\frac{l}{3} & \frac{l}{6}\\ \frac{l}{6} & \frac{l}{3}\end{bmatrix};\quad \mathbf{f}^{(e)}=\begin{bmatrix}\frac{l}{2}\\ \frac{l}{2}\end{bmatrix}r(x,t)$$

The lumped scheme is based on the spatial interpolation functions expressed in the forms [7]:

$$N_{j-1}^{*}=1-H\left(s-\frac{1}{2}\right);N_j^{*}=H\left(s-\frac{1}{2}\right);\quad N_j^{*}=H\left(s-\frac{1}{2}\right)$$

The Heaviside function H(x) is defined as:

$$H(x)=0 \quad \text{if } x<0$$

$$H(x)=1 \quad \text{if } x\geq 0$$

where s = distance from node j − 1; l is the length of the element.

The matrixes for the lumped scheme of Equation (6) can be estimated in the form:

$$\mathbf{A}^{(e)}=\frac{1}{2}\begin{bmatrix}l & 0\\ 0 & l\end{bmatrix}$$

The matrix $\mathbf{B}^{(e)}$ and vector $\mathbf{f}^{(e)}$ remain the same as the linear scheme.

For the entire domain containing the elements, Eq. (6) has the form: