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Assessing solution quality and computational performance in the long-term generation scheduling problem considering different hydro production function approaches

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A B S T R A C T

The long-term generation scheduling (LTGS) problem aims at finding a generation policy that minimizes an objective function over a multi-year planning horizon. A crucial aspect of this problem is the Hydropower Production Function (HPF), which relates power with head, turbined outflow, and efficiency of the generating units. Given that the LTGS is a large-scale stochastic optimization problem, the HPF is modeled in a simplified manner. However, considering the high-performance computers currently available and the recent advances in stochastic optimization algorithms, it is possible to enhance the HPF modeling to use the energy resources more efficiently. This paper proposes a piecewise linear model of HPF that considers the plant generation as a function of the volume and the total outflow. Unlike previous works, the HPF also considers the (nonlinear) efficiency function of each generating unit. The paper also presents a comparison between the proposed HPF and a one-dimensional HPF known as constant productivity. The generation policy and the computational burden are analyzed using an optimization-simulation process based on Stochastic Dual Dynamic Programming algorithm. The computational tests use data of a large-scale electrical power system, which corresponds to about 90% of the Brazilian system.

1. Introduction

The long-term generation scheduling (LTGS) problem seeks to find a generation policy that minimizes an objective function over a multi-year planning horizon. Usually, the objective function is given by the expected value or/and a risk measure of the operational cost (related to the thermal generation and deficit). The main constraints are the demand requirements, the water balance of the reservoirs and operating limits of generating units. Regardless of the market regulatory framework (i.e., liberalized markets or monopolistic producer), LTGS is very important for the sustainability in a hydro dominated system since it provides an insight about the expected value of the water stored in the reservoirs. This value defines the optimized operation in a monopolistic framework or it is used as information for setting the price and quantity of generator's bids in a liberalized market. In systems with predominately hydro resources (e.g., Brazil, Norway, New Zealand), the LTGS is usually represented by a multistage linear stochastic optimization problem [1]. Inflows are included in this problem via scenario tree. Due to the high dimensionality of the problem, Stochastic Dual Dynamic Programming (SDDP) [2] is the most used solution strategy. Roughly speaking, the SDDP applies forward and backward steps of Nested Decomposition [1] combined with sampling strategies [3]. At the end of the iterative process, SDDP provides future cost functions (FCFs), which quantify the water's future value as a function of reservoir storage and/or past inflows for each stage of the planning horizon [4]. These FCFs are used to obtain generation policies, as well as marginal costs, deficit probabilities, expected volumes, etc.

A crucial aspect of the LTGS is the hydropower production function (HPF). It is well known that the HPF of a single generating unit is given by the product of efficiency, net head, and turbined outflow. From a mathematical point of view, HPF is a nonlinear (and nonconvex) function with discontinuities related to the forbidden zones [5]. Traditionally in LTGS, the HPF modeling is very simple due to the computational burden involved in solving this large-

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Nomenclature

Sets and symbols

$\mathscr{S}$	set of subsystems
$\mathscr{R}$	set of hydro plants
$\mathscr{F}$	set of thermal plants
$\mathscr{D}$	set of deficits
$\mathscr{L}$	set of load levels
$\mathscr{R}^{\mathscr{S}}$	set of hydro plants located in subsystem s
$\mathscr{F}^{\mathscr{S}}$	set of thermal plants located in subsystem s
$\mathscr{O}^{\mathscr{S}}$	set of subsystems connected with subsystem s
$\mathscr{M}^{r}$	set of hydro plants upstream from plant r
$\mathscr{R}^{\mathscr{A}}$	set of hydro plants with monthly regularization capacity
R^{B}	set of run of river hydro plants
$\mathbb{E}[\bullet]$	expected future cost

Indexes

t	index of stages
u, z	indexes of HPF piecewise linear approximation
i	index of generating unit in the hydro plants
k	index of discretization in v dimension
j	index of discretization in d dimension
c	index of future cost function piecewise linear approximation
g	index associated with the points used to calculate the RMSE

Variables

$pu_{i,r}$	power of unit i and hydro plant r (MW)
$h_{i,r}$	net head of unit i and hydro plant r (m)
$\eta_{i,r}$	hydraulic efficiency of the unit i and hydro plant r
$w_{i,r}$	turbined outflow of unit i and hydro plant r (m^3/s)
$v_{t,r}$	volume in the hydro plant r in the beginning of stage t (hm^3)
q_r	turbined outflow in the hydro plant r (m^3/s)
s_r	spillage in the hydro plant r (m^3/s)
d_r	total outflow in the hydro plant r (m^3/s)
$d_{t,l,r}$	total outflow in the hydro plant r, load level l and stage t (m^3/s)
ph_r	output power of the hydro plant r (MW)
$ph(v^k, d^j)$	plant output power considering fixed values of volume (v^k) and total outflow (d^j) (MW)
$ph_{t,l,r}$	power of hydro plant r, load level l and stage t (MW)
$pf_{t,l,f}$	power of thermal plant f, load level l and stage t (MW)
$pd_{t,l,d}$	power of deficit d, load level l and stage t (MW)
$po_{t,l,o}$	power interchange of subsystem o, level l and stage t (MW)
α	expected future cost (BRL)
x_t	decision variable at stage t
Q_t	optimal value of linear program at stage t
ξ_t	random vector at stage t
N_t, H_t, b_t	data at stage t

Constants

$\mathrm{A}_{k,r}$	coefficients of the forebay elevation function of plant r, where $k = 0, \ldots, 4$ (m/m^k)
$\mathrm{B}_{k,r}$	coefficients of the tailrace elevation function of plant r, where $k = 0, \ldots, 4$ (m/m^k)
$\mathrm{C}_{r,i}$	penstock loss factor of unit i and hydro plant r ($\mathrm{s}^2/\mathrm{m}^5$)
$\mathrm{D}_{k,i,r}$	coefficients of the turbine hydraulic efficiency function of unit i and plant r, where $k = 0, \ldots, 5$
$\mathrm{E}^{z}_{k,r}$	coefficient associated with HPF hyperplane z of hydro r, where $k = 0, \ldots, 3$
$\mathrm{F}^{u}_{k,r}$	coefficient associated with HPF hyperplane u of hydro r, where $k = 0, \ldots, 2$
ρ_r	productivity of the hydro plant r [$\mathrm{MW}/(\mathrm{m}^3/\mathrm{s})$]
K_0	conversion factor to convert water discharge (m^3/s) in volume (hm^3)
CF^{f}	vector with the incremental costs of thermal generation f (BRL/MWh)
CD^{d}	vector with the incremental deficit d (BRL/MWh)
$\mathrm{PL}_{t,l,s}$	demand in subsystem s, load level l and stage t (MW)
V^0_r	initial volume of reservoir r (hm^3)
$\overline{\mathrm{W}}_i$	maximum turbined outflow of the hydro unit i (m^3/s)
$\overline{\mathrm{PU}}_i$	maximum power of the hydro unit i (MW)
$\overline{\mathrm{PD}}_d, \overline{\mathrm{PH}}_r, \overline{\mathrm{PT}}_f$	maximum values of deficit d, power of hydro plant r, and power of thermal plant f, respectively (MW)
$\underline{\mathrm{PO}_o}, \overline{\mathrm{PO}}_o$	minimum/maximum power interchange of subsystem o (MW)
$\underline{\mathrm{V}_r}; \overline{\mathrm{V}}_r$	minimum/maximum volume of reservoir r (hm^3)
$\overline{\mathrm{D}}_r$	maximum total outflow of hydro plant r (m^3/s)
Π^{c}_{r}	dual coefficient associated with reservoir r and hyperplane c of the future cost function ($\mathrm{BRL}/\mathrm{hm}^3$)
Z^{c}	parameter associated with hyperplane c of the future cost function (BRL)

scale stochastic programming problem and the SDDP convexity requirements [6]. In order to obtain a simplified HPF model, two requirements should be considered: (*i*) the HPF model must be concave, to ensure convergence of the optimization algorithm; (*ii*) it must have as few constraints and variables as possible since usually many inflow scenarios are used in the SDDP.

In this context, the simplest HPF modeling in LTGS is the one based on the Equivalent Energy Reservoir (EER) model [4] [7], [8]. Although the previous requirements (*i*) and (*ii*) are met, the quality of the policy obtained by the EER can be very poor since the aggregation of several reservoirs neglects the individual constraints of the hydro plants. In fact, there is also the risk that the policy will not even be feasible [9]. In turn, when considering individual plants modeling, the most common approach is known as constant productivity [10], where the HPF is a linear function of the plant turbined outflow. In this case, the individual operational characteristics of the Plants[1] are considered and only one variable is used in the HPF model. However, the main disadvantage lies in disregarding the head effect, and several works have tried to deal with this issue in basically two ways: (i) correcting the constant (productivity) as a function of the net head; (ii) using a linear (or piecewise linear) model as a function of volume.[2] For example, paper [11] uses linear regression techniques to eliminate nonlinearities associated with the net head. Moreover, to avoid nonconvexities, in Ref. [12] the HPF is represented as a piecewise linear function of the turbined outflow and, to include the head effects, a heuristic based on an economic sensitivity in the objective function

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[1] Unlike EER, the hydraulic balance constraints and operational limits of the reservoirs are explicitly represented in LTGS.

[2] And other variables that affect the net head.