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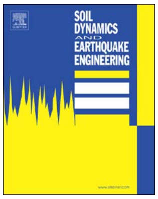

Parametric estimation of dispersive viscoelastic layered media with application to structural health monitoring

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ABSTRACT

We present a sequential Bayesian estimation method to estimate the material parameters that govern the one-dimensional propagation of shear waves through continuous, layered, viscoelastic solids. While the proposed estimation method is generic, namely, can be applied to waveform inversion problems that satisfy the above conditions, we here employ it for system identification of building structures. We approximate the linear-elastic response of building structures subjected to low-amplitude earthquake base excitations by a multilayer dispersive shear beam model with Kelvin-Voigt material subjected to vertically propagating shear waves. Utilizing the proposed sequential Bayesian estimation method, we sequentially update the probability distribution function of the unknown parameters to reduce the discrepancies between the estimated and measured frequency response functions. We next verify and validate the performance of the proposed estimation method and investigate the limitations of the presented structural system identification approach using two case studies. In the first case study, we use the simulated structural response of a three-dimensional 52-story building model subjected to bi-directional low-amplitude ground shakings. We estimate the frequency-dependent phase velocity and damping ratio, as well as the mass distribution along the building height. Then, we verify the structural damage detection and localization capabilities of the presented system identification approach by comparing the wave model parameters estimated from simulated response of undamaged and damaged structural models. In the second case study, we use data measured from a shake table experiment on a full-scale five-story reinforced concrete building specimen, where the estimated wave model parameters capture the progressive structural damage in the test specimen. The validation studies suggest that the sequential Bayesian estimation method based on viscoelastic dispersive wave propagation can be used for system and damage identification of building structures.

1. Introduction

The linear-elastic response of a building structure subjected to an earthquake base excitation can be approximated by modeling the propagation of seismic waves through a continuous, layered, viscoelastic solid. The velocity of shear waves propagating through the building can be estimated using impulse response functions (IRFs) (e.g., [1–6]). Since the shear wave velocity is related to the lateral stiffness of the building structure, several studies were able to detect structural damage in terms of loss of effective lateral stiffness by comparing the estimated shear wave velocities from the pre- and post-damage IRFs (e.g., [7–9]).

Among others, Rahmani and Todorovska [10] identified the shear wave velocity profile of an equivalent multilayer shear beam model, representing the Millikan Library building, using IRFs estimated from the measured response of the building to a small earthquake. They used a least squares method to minimize the discrepancies between the main pulses of the estimated and measured IRFs, and to identify the wave velocity profile along the building height. In another study, Ebrahimian et al. [11] estimated the frequency-dependent phase velocity of the vertically propagating shear waves in a building structure. They used an equivalent Timoshenko beam model and estimated its phase velocities by measuring the wave travel times from IRFs derived from band-passed-filtered responses at various frequency ranges. This study provided a nonparametric method for identification of dispersive shear wave propagation in building structures. Lastly, in recent studies, Ebrahimian and Todorovska [12,13] presented a parametric identification method to identify the frequency dependent phase and group velocities of vertically propagating waves in building structures. They used a least squares method to estimate the stiffness parameters of an equivalent multilayer Timoshenko beam model, which accounts for dispersion due to bending deformation, by minimizing the discrepancies between the main pulses in the estimated and measured IRFs.

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In this paper, we present a sequential Bayesian estimation method for parametric estimation of wave dispersion in a continuous, multilayer, viscoelastic solid. In contrast to the aforementioned studies, which used the IRFs to estimate the wave velocity, we formulate our estimation approach in the frequency domain – i.e., using the frequency response functions (FRFs). It should be noted that the sequential Bayesian estimation method is general, and can be applied to frequency-domain or time-domain waveform inversion problems. However, we noticed an improved estimation performance when the presented estimation method is used in the frequency domain. The estimation method is validated in this study using two building system and damage identification case studies: the first is based on the numerically simulated data obtained from linear-elastic structural models, and the second is based on the data measured from a full-scale shake table experiment. In these case studies, the buildings are identified as multilayer dispersive shear beam model with Kelvin-Voigt material, in which each layer represents a story or group of stories. We next propose empirical functions to characterize the dispersion of phase velocity and damping ratio at each layer. We estimate the parameters characterizing the frequency-dependent phase velocity, damping ratio, and mass density of each layer, and quantify their estimation uncertainties, by minimizing the discrepancies between the measured and predicted FRFs.

The novelty of this study is threefold: (i) we propose a sequential Bayesian estimation method using FRFs for waveform inversion to estimate the unknown wave propagation parameters and to quantify their estimation uncertainties; (ii) instead of estimating shear wave phase velocity and damping ratio for different frequency ranges, we propose closed form empirical dispersion functions to characterize the phase velocity and damping ratio as a function of frequency; we then estimate the function parameters through the Bayesian estimation method; and, (iii) we provide a methodology that can be used for a wide range of waveform inversion problems, from soil response inversion of downhole geotechnical array recordings to structural system and damage identification of building structures.

This study has been motivated by the Community Seismic Network (CSN) project [14–17], which is a network of low-cost micro-electromechanical systems (MEMS) accelerometers that have been used to densely instrument buildings and free field locations in the greater Los Angeles area. Most CSN-instrumented buildings are equipped with one or two triaxial accelerometers per floor. The primary product of the network is the measured structural response time histories on a floor-by-floor spatial scale, during and after earthquakes. The complementary product of this network would be robust and computationally efficient methods for structural damage identification that not only detect but also localize and quantify damage in the building rapidly after an earthquake. This would provide a proxy indicator of building damage that can inform emergency response activities mobilized after the event. This study provides a method that can be used towards this objective.

2. Problem statement

As mentioned above, we model the building structure as a continuous, non-homogeneous (along height), viscoelastic solid, subjected to vertically propagating shear waves imposed as prescribed displacement boundary conditions at the base (Fig. 1). The frequency-dependent phase velocity and attenuation of seismic energy at different wavelengths, together with the geometric and inertial properties of the building characterize its dynamic response. The objective of this study is to identify the building system by estimating parameters that characterize the propagation of shear waves through the system.

To achieve this goal, we estimate the frequency response functions (FRFs) of each floor's absolute acceleration response with respect to the base excitation, using a seismic interferometry approach [18] based on the spectral analysis method [19]. The FRFs estimated from the measured structural responses (i.e., measured FRFs) are then compared with FRFs predicted using analytical models for one-dimensional wave propagation in a multilayer dispersive shear beam model with Kelvin-Voigt material. Through a sequential Bayesian estimation approach, we estimate the parameters characterizing the phase velocity and damping ratio as a function of frequency, as well as the homogenized mass density of the building. These parameters will hereafter be referred to as *wave model parameters*. These parameters are used for structural system and damage identification in the ensuing sections of this paper.

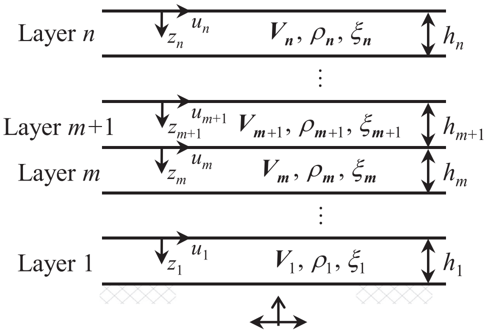

Fig. 1. Multilayer Kelvin-Voigt shear beam model on a rigid base.

Each layer of the multilayer Kelvin-Voigt model is an idealized homogenized story or group of stories of the building structure. The premise of our study is that estimating the wave model parameters before and after a damage-inducing earthquake can provide information about the extent and location of the damage. More specifically, by interpreting the reduction in the phase velocity as a permanent loss of lateral stiffness at the story level (or group of stories) of the building structure, we can detect and localize the structural damage (e.g., [7–9]).

3. Parametric estimation of seismic wave dispersion in building structures

3.1. Multilayered Kelvin-Voigt model for one-dimensional shear wave propagation

The analytical solution for shear wave propagation in a multilayer Kelvin-Voigt shear beam was first introduced by Gilbert and Backus [20]. This solution is similar to the Thompson [21] and Haskell [22] propagator matrix approach and is implemented in the computer program, SHAKE [23]. Although the solution has already been presented in the literature (e.g., [6,24] among others), it is briefly reviewed here to ensure the completeness of the discussion.

The solution to the vertically propagating SH-wave in a layered shear beam (Fig. 1) for a harmonic wave of frequency f can be expressed as

$$u(z,\, t) = A\ \ e^{i(2\pi f\ t + k^* z)} + B\ \ e^{i(2\pi f\ t - k^* z)} \tag{1}$$

in which A, B = constants representing the amplitude of the waves travelling in and $+\ z$ (upward and downward) directions, respectively, and $k^* = 2\pi f / V^*$ is the complex wave number. The term $V^* = V\sqrt{1 + 2i\xi}$ is the complex shear wave velocity and $\xi = \pi\eta f / G$ is the damping ratio, where η denotes the viscosity and G denotes the shear modulus of the Kelvin-Voigt material model [24]. By introducing a local coordinate system for each layer as shown in Fig. 1, the displacement and force continuity condition at each layer interface yields the following two equations.

$$u_{m+1}(h_{m+1},\, t) = u_m(0,\, t) \Rightarrow A_{m+1} e^{ik^*_{m+1} h_{m+1}} + B_{m+1} e^{-ik^*_{m+1} h_{m+1}} = A_m + B_m \tag{2}$$