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Simplified formulas for the seismic bearing capacity of shallow strip foundations

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ABSTRACT

The seismic bearing capacity of shallow foundations is affected by inertia forces acting both on the structure and in the supporting soil. Even though the former have been recognised to play often the major role, by increasing the horizontal load and the overturning moment transferred to the foundation, both of them must be taken into account in the seismic design of foundations. Using a pseudostatic approach and based on the upper bound theorem of limit analysis, a comprehensive set of formulas is derived for the computation of the seismic bearing capacity of strip footings resting on cohesive-frictional and purely cohesive soils. Results are given in terms of: (i) reduction coefficients for the Terzaghi's equation of the vertical bearing capacity and (ii) ultimate failure envelopes in the space of normalised loading variables. These formulas extend to more general conditions other literature results, allowing to take into account easily the effects of inertia forces acting both on the superstructure (load inclination and eccentricity) and into the foundation soil. The reliability of the proposed equations, suitable for the design practice, is verified through a thorough comparison with other rigorous and approximate solutions.

1. Introduction

Many studies on the seismic bearing capacity of shallow foundations have shown that inertia forces acting on the structure and in the supporting soil tend to reduce the bearing capacity under seismic conditions. Most works on this topic have been carried out with a pseudostatic approach [18,22,23,25,3,29,30,32,34,35,5,6,9], by adopting: (i) different methods (numerical or theoretical) and theories (limit equilibrium, limit analysis or method of characteristics); (ii) different constitutive assumptions for the soil (purely frictional, purely cohesive or cohesive-frictional); (iii) different hypotheses on the inertia forces on the soil (with or without the vertical component) and (iv) on the structure (equal to or a fraction of those acting on the soil).

Despite the fact that structure inertia has been recognised to play often the major role in reducing the seismic bearing capacity of shallow foundations, recent studies have highlighted possible situations in which even the effects associated to soil inertia can have a significant relevance, in the case of either frictional [22,6] or purely cohesive [24] soils. Moreover, most design codes recommend to take into account the effects of soil inertia in the seismic design of such systems (*e.g.*: [10]).

Going to the design practice, the bearing capacity of shallow foundations under general loading is usually evaluated by means of simple approaches, neglecting any possible soil-structure interaction effect. In this context, codes and guidelines make use of closed form expressions for the bearing capacity, given in the form of either the classical Terzaghi's formula [1,17] or complete three-dimensional failure envelopes [10]. With this respect, only few works in the literature provide empirical formulas including inertia forces both on the structure and into the soil.

As far as spread footings on cohesive-frictional soils are concerned, Budhu and Al-Karni [3], Paolucci & Pecker [23] and Cascone et al. [5] provide reduction factors for the vertical bearing capacity. However, Budhu and Al-Karni [3] consider the same accelerations into the soil and the structure; Paolucci & Pecker [23] do not contemplate the effects of the structure inertia on the N_c and N_q bearing capacity factors, while Cascone et al. [5] refer only to the effects of the seismic action on the N_γ term, thus resulting in a limited applicability of the proposed formulas. Only very recently, Cascone & Casablanca [6] proposed empirical expressions for the reduction coefficients, derived from the best fit of numerical results.

On the other hand, no reduction coefficients are available for the case of shallow foundations on purely cohesive soils, while, in this case, an approximate equation of the failure envelope was proposed by Faccioli et al. [11], based on results of limit analysis [24,25].

This work aims to provide a comprehensive set of empirical equations for the evaluation of the seismic bearing capacity of shallow strip foundations resting on a homogeneous layer of either cohesive-frictional or purely cohesive soil. Moreover, the relative merits of structure

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and soil inertia in the reduction of the bearing capacity are discussed. To this end, following a pseudostatic approach, the upper bound theorem of limit analysis is used, by modelling the soil as an elasto-plastic material with a Mohr-Coulomb and Tresca yield criterion respectively.

Given the inherent uncertainties in the definition of the parameters involved in a bearing capacity calculation, related to both the geotechnical soil model and the earthquake input motion, the simplicity of the empirical equation is by all means a key ingredient when suggesting formulas to be used in the design practice. This is indeed one of the underlying ideas of this work, where, after a thorough comparison of the upper bound results with other literature data, simple formulas are proposed for the reduction coefficients of the Terzaghi's equation, partly incorporating the empirical equations provided by Hansen [14], widely used in the static design practice. Moreover, the same reduction coefficients are used to construct three-dimensional failure envelopes for shallow strip foundations under pseudostatic loading.

Neither the effects of pore water pressure nor the reduction of the shear strength of the soil due to seismic effects are taken into account. Different inertia forces are considered on the structure and into the soil.

2. Problem definition and theoretical framework

Fig. 1 shows the problem under examination, consisting of a shallow strip foundation (width B, embedment depth D) resting on a homogeneous soil (unit weight γ, friction angle ϕ, cohesion c). The foundation is subjected to an inclined and eccentric load, including both the static and inertia forces transmitted by the superstructure. The load is defined by its vertical component V, its horizontal component $H = V \tan\beta$ and an overturning moment $M = V e$, where β and e are the angle of inclination and the eccentricity, respectively. The inertia forces into the soil are introduced through the pseudostatic coefficients k_h and k_v, acting in the horizontal and vertical direction, respectively. The soil above the foundation level is replaced by a shear and normal stress distribution proportional to the dead weight of the lateral soil, $q = \gamma D$.

The load eccentricity is taken into account only indirectly, by assuming a reduced effective width $B' = B\text{-}2e$, in agreement with the Meyerhof's suggestion [20]. This strategy, often adopted in the literature to reduce the complexity of the problem at hand, provides a good approximation of the collapse load for shallow footings resting both on sand [19] and clay [15,36].

According to limit analysis, an upper-bound of the exact collapse load can be obtained by equating the power of external forces (P^{ext}) to the power of internal dissipation (P^{int}), computed with reference to a kinematically admissible collapse mechanism. Following Dormieux & Pecker [9], a non-symmetrical Prandtl's mechanism is examined, characterised by two rigid wedges connected by a log-spiral plastic zone, the latter reducing to a circle for a pure cohesive material. The geometry of the failure mechanism, completely defined by the two angles ρ and ψ, together with the assumed kinematic field, is given in Fig. 1(c).

The reader may refer to Chen & Liu [7] for a thorough dissertation on the upper-bound theorem of limit analysis, while its application to the specific mechanism considered herein is detailed in Appendix A and B for a cohesive-frictional soil (M-C yield criterion) and a purely cohesive soil (Tresca yield criterion) respectively.

The average limit load corresponding to the assumed failure mechanism can be expressed as:

$$q^*_{\lim}(\rho, \psi) = \frac{1}{2}\gamma B' N^*_{\gamma E} + cN^*_{cE} + qN^*_{qE} \tag{1}$$

where $N^*_{\gamma E}$, N^*_{cE} and N^*_{qE} are functions of the geometry of the failure mechanism, material properties, load inclination, and pseudostatic soil accelerations. The upper-bound estimate of the bearing capacity is given by:

$$q_{\lim} = \min_{\mathbf{H}(\rho,\psi)\le 0} q^*_{\lim}(\rho, \psi) \tag{2}$$

where $\boldsymbol{H}(\rho,\psi)$ is the vector of physical and/or geometrical constraints. Eq. (2) can be solved by numerical minimization and the results given in standard form as:

$$q_{\lim} = \frac{1}{2}\gamma B' N_{\gamma E} + cN_{cE} + qN_{qE} \tag{3}$$

where $N_{\gamma E}$, N_{cE} and N_{qE} are the seismic bearing capacity factors.

Based on the best fit of rigorous upper bound numerical solutions, the following sections provide a comprehensive set of simplified formulas for the seismic bearing capacity factors of shallow strip foundations. In order to simplify the structure of the empirical equations, the vertical pseudostatic coefficient is not taken into account in their derivation ($k_v = 0$), thus implicitly neglecting any contribution of the vertical soil acceleration. This assumption if often introduced when dealing with the seismic stability of geotechnical systems, including shallow foundations [16,29–31], based on the fact that the vertical acceleration is generally out of phase with and has a different frequency content than the horizontal component, with the corresponding peak

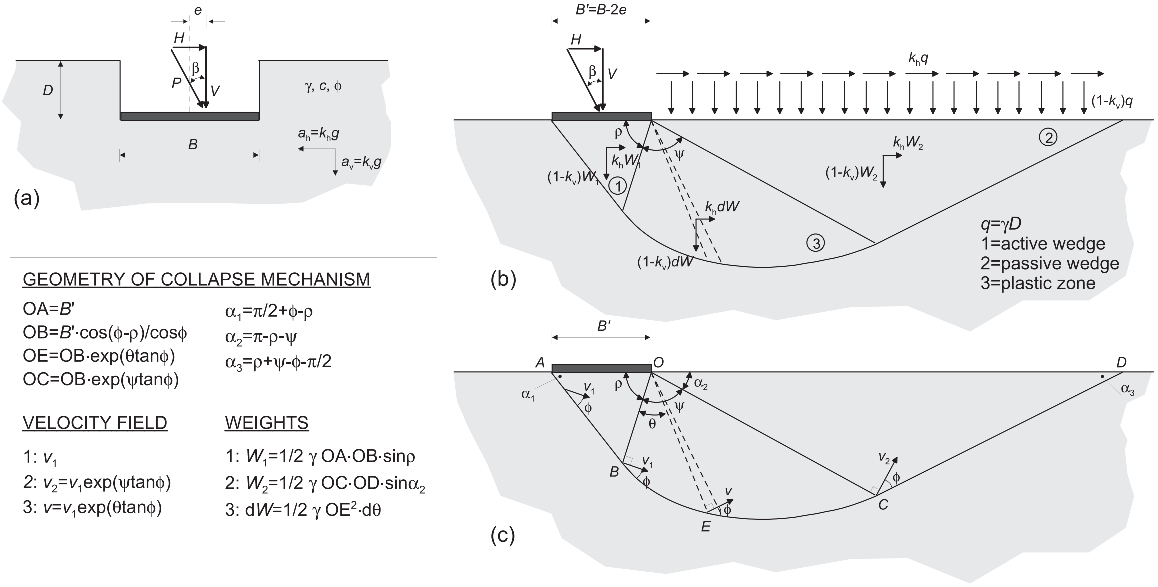

Fig. 1. Shallow strip foundation on homogeneous Mohr-Coulomb soil: (a) geometry and load configuration, (b) failure mechanism and (c) velocity field.