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Wave-induced dynamic response of saturated multi-layer porous media: Analytical solutions and validity regions of various formulations in non-dimensional parametric space

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ABSTRACT

In this paper, dynamic response of saturated-layered porous media under harmonic waves is evaluated through a semi-analytical solution. The coupled differential equations governing the dynamics of saturated or nearly saturated porous media such as soils containing all the inertial terms of solid and fluid phases are presented for a multi-layer system. Possible simplifications of the equations which are called *formulations* are introduced based upon the presence of inertial terms associated with the phases. The semi-analytical solutions to the response of multiple layers for all the formulations are presented in terms of pore water pressure and stress variations considering a set of non-dimensional parameters and their respective ratios. Validity of the formulations is presented in a non-dimensional parametric space. The maximum discrepancies in the pore pressure response of the formulations leading to validity regions are illustrated for typical dynamic problems. Subsequently, the effects of layering and drainage conditions on these regions are also presented. The proposed semi-analytical solution may be served as a benchmark one for validating the coupled numerical solutions, which can be used to deal with real scientific and geo-engineering problems in the emerging field of computational geomechanics.

1. Introduction

Saturated or nearly saturated porous media are typically found in fields such as soils, various construction materials, biological systems etc. These regions generally consist of multiple layers that are either naturally deposited or artificially placed essentially altering their physical and/or engineering properties. Therefore, it is necessary to consider these systems as continuous saturated multi-layered porous areas and that the analyses requiring the true behavior of these systems need to take into account the simultaneous processes of coupled flow (of pore fluid, generally water) and deformation (of the solid skeleton, generally grainy particle composition). As far as engineering and scientific solutions to this coupled problem, there have been several analytical, semi-analytical and numerical approaches in the recent decades. Disregarding the earlier studies, Madsen [1] and Yamamoto et al. [2] were the first to develop analytical solutions for the wave-induced response of seabed using the theory of coupled flow and deformation [3]. More complete formulations of the problem, including the wave-seabed interactions, the inertial effects and vertical variation of soil properties, were later developed by Yamamoto [4] and Yamamoto and Schuckman [5]. Mei and Foda [6,7] studied the wave-induced response of seabed by developing a general approximation and the associated existence of a boundary layer system. Okusa [8] developed a solution to the wave-induced response of saturated seabed. Siddharthan [9] studied the wave-induced displacements in seafloor sands. Later, a general semi-analytical method for a quasi-static response was developed by Rahman et al. [10]. Gajo and Mongiovi [11] developed an analytical solution for the transient response of one-dimensional (1-D) saturated system and Valliappan et al. [12] developed a solution to two-dimensional (2-D) dynamic consolidation in frequency domain. More recently, Jeng and his colleagues [13–15] developed various analytical solutions including the inertial effects and wave non-linearity on the response of a single layer porous medium. It should be noted that in all these works, a linear elastic constitutive relation was used to model the deformation of soil skeleton along with simplified geometrical domains. For the engineering problems in which the factors such as the dynamic soil-structure interaction, the actual geometrical shape of a structure and the nonlinear behavior of the foundation and its surrounding soils cannot be solved with these analytical methods. For this reason, advanced numerical methods have been developed to solve not only the wave radiation problems [16,17], but also a wide range of scientific and engineering problems with finite and infinite domains [18–20]. There are also recently

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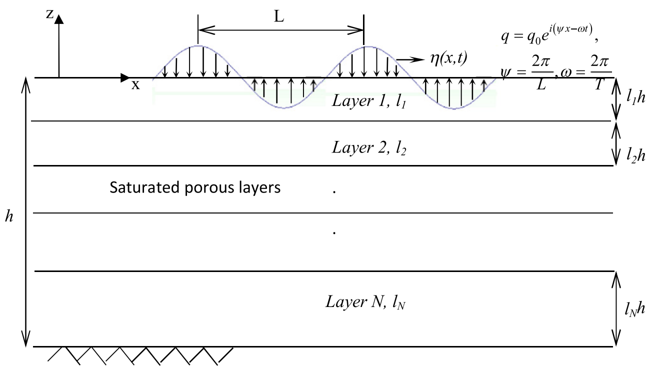

Fig. 1. Two dimensional multi-layer porous medium under harmonic load.

published research studies in this field which make it possible to numerically solve the problem of coupled flow and deformation considering the elastoplastic behavior under wave loading [21,22]. Such numerical solutions have been validated against the physical data sets from experiments or field measurements. This study, however, provides a semi-analytical approach to evaluate the wave-induced seabed response assuming the constitutive behavior as linearly elastic. Since the upper crust of the Earth can be mathematically treated as a system consisting of saturated-layered porous media [23], analytical solutions of this study can also be used to get some understanding of seismic behavior within the Earth's crust. From this point of view, the theoretical results of this study can enrich the contents of the emerging computational geoscience field [24] in a broader sense.

Generally, most of these studies have involved a single soil layer in the domain [25] and so multiple layer solutions have been limited [26,10,27]. Mesgouez and Mesgouez [28] developed a semi-analytical approach to study wave propagation in a multi-layer poro-visco-elastic system due to transient loads. Shan et al. [29] similarly developed exact solutions for 1-D transient response of a single layer of saturated porous media. Subsequently, Ai and Zeng [30] analyzed Biot's consolidation of multi-layer soils subjected to non-axisymmetric loading in an arbitrary depth and recently, Ulker [31] developed analytical and numerical solutions to the dynamic response of a saturated seabed. However, that was a limited 1-D solution. It can be seen from the literature that the solutions proposed thus far have considered either a single layer with or without inertial terms or a few layers without the inertial terms for 1-D. In this study, the aim is to resolve the multi-layer dynamic response with all the inertial terms included in the solution. It is also intended to provide the engineers and researchers with the necessary tools to decide what formulation should be used to evaluate the dynamic response of porous media in a given geo-engineering problem.

In this study, analytical solutions are proposed considering three mathematical formulations to evaluate the dynamic response of a multi-layer saturated porous media utilizing the Biot's formulation [3,32,33]. For that, the constitutive relation of the solid phase is taken linear isotropic elastic. Hence only the oscillatory porous media response is considered, excluding the residual (build-up) response. Depending on the inclusion of inertial terms in the governing equations, three possible formulations are introduced, namely (i) fully dynamic (FD) formulation requiring the inertial terms associated with both the motion of solid skeleton and that of pore fluid; (ii) partly dynamic (PD) formulation with the inertial terms associated with only solid skeleton and, (iii) quasi-static (QS) formulation where all the inertial terms are neglected. The main objective of this study is to derive semi-analytical solutions to all three formulations for a system of a number of saturated porous layers with distinct

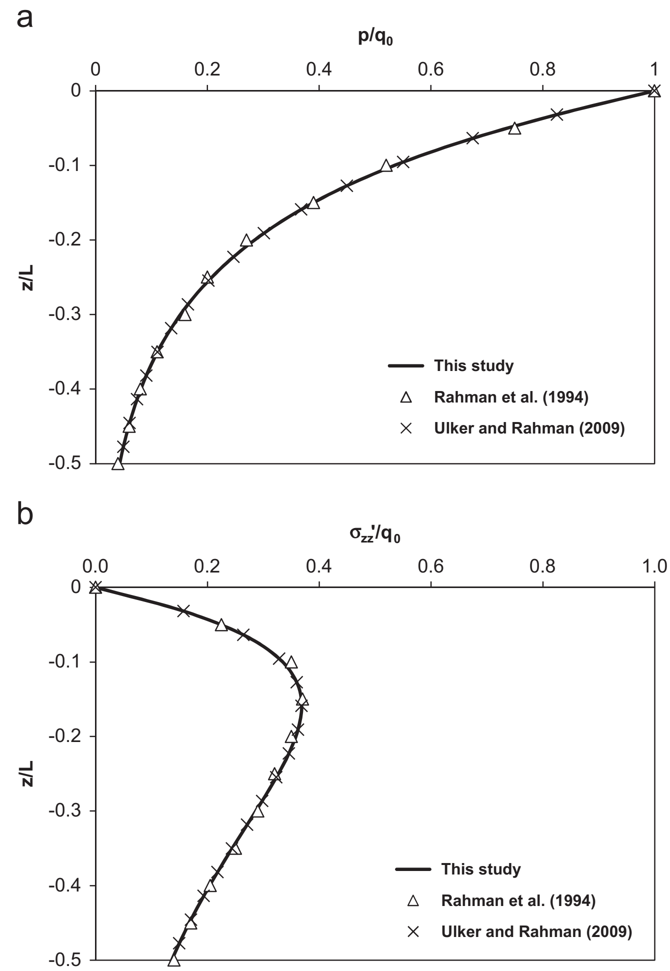

Fig. 2. (a) Pore pressure, (b) shear stress response of QS formulation for a single layer, $m=1$, $T=10$s, $k_z=0.01$ m/s, and $G=25$ MPa.

physical properties. It is also necessary to develop regions of applicability of formulations in terms of non-dimensional parameters to be utilized in solving a specific engineering or a scientific problem in a particular field consisting of porous media. The derived analytical solution may be served as a benchmark one for extending the dynamic infinite element [20,17] and validating the coupled method of finite and dynamic infinite elements, which can be used to deal with problems in the emerging computational geomechanics field. In addition, the presented solution can also be used to have an understanding of whether an FD model, a PD model or a QS model should be used in the solution of an engineering problem.

2. Governing equations

The three basic laws; (i) constitutive relation for the stress–strain relationship, (ii) momentum balance providing the equilibrium equations of solid and liquid phases, and (iii) mass balance equation for the fluid phase, are necessary to develop a system of equations defining the fully dynamic response of any two-phase porous medium. Porous media can be viewed as two phase materials considering the solid grains with different size and shape, and a viscous fluid (generally water) allowed to flow freely through the pores of the medium. The air phase is assumed to be fully dissolved in the fluid constituting a single compressible fluid. This section briefly summarizes the governing equations proposed first by [3] who later included dynamic terms [32,33].