HOSTED BY
ELSEVIER

Available online at www.sciencedirect.com
ScienceDirect

Soils and Foundations xxx (2018) xxx–xxx

SOILS AND FOUNDATIONS

www.elsevier.com/locate/sandf

# Bifurcation prediction of shear banding in sand with non-coaxial critical state model considering inherent anisotropy

Maosong Huang*, Zhouquan Chen, Xilin Lu

*Department of Geotechnical Engineering, Tongji University, Shanghai 20092, China*
*Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education, Tongji University, Shanghai 200092, China*

Received 17 March 2017; received in revised form 5 February 2018; accepted 15 February 2018

## Abstract

In view of phenomenological observations, the anisotropic state variable accounting for the effect of inherent fabric anisotropy, which can describe the microstructural characteristics of sand, is thought to influence the critical state of sand. Based on revised forms of the anisotropic state variable, the form for the anisotropic critical state line function is revised and the non-monotonically anisotropic effects are incorporated into the strength parameter in the re-adjustment of the elements of the model. The anisotropic plasticity model parameters of Toyoura sand are re-calibrated with triaxial compression and extension tests having various deposition directions. In comparison to the experiments on Toyoura sand, it is shown that the model can simulate the anisotropic stress-strain relationship reasonably well. Then, bifurcation analyses are conducted to predict the onset of strain localization with the corresponding non-coaxial anisotropic model. The results show that the modeling of shear band formation based on the non-coaxial model is in agreement with the tests considering the anisotropic characteristic.

*Keywords:* Fabric anisotropy; Critical state sand model; Bifurcation analysis; Non-coaxial plasticity theory; Strain localization

## 1. Introduction

Strain localization associated with the occurrence of shear banding is a common feature of soils that have failed in geotechnical structures, such as slopes, embankments and foundations. Experimental results have revealed that localized deformation is always observed along with the overall reduction in strength in material stress-strain behavior. This suggests that predicting the initiation of shear banding is important to geotechnical applications. However, it is also noted that the influence of the anisotropy of the materials on the failure and shear band formation is not sufficiently taken into account.

In the past, the bifurcation theory proposed by Rudnicki and Rice (1975) indicated that constitutive models play an important role in predicting the initiation of shear banding. Predictions of the onset of shear banding based on a conventional coaxial plasticity model are shown to result in poor results under a plane strain condition (Rudnicki and Rice, 1975; Papamichos and Vardoulakis, 1995; Vardoulakis and Graf, 1985; Molenkamp, 1985), while those based on a corresponding non-coaxial model are more approximate to the test results. Such a non-coaxial modeling platform is generically developed by introducing a vertex-like structure to the basic coaxial constitutive model, which leads to a non-coaxial plastic flow rule. An improvement in the predictions based on the non-coaxial model has been validated in bi-axial tests using the Drucker-Prager model (Papamichos and Vardoulakis, 1995) and the Mohr-Coulomb model (Bardet, 1991), but

---

Peer review under responsibility of The Japanese Geotechnical Society.

* Corresponding author at: Department of Geotechnical Engineering, Tongji University, Shanghai 20092, China.
*E-mail address:* mshuang@tongji.edu.cn (M. Huang).

https://doi.org/10.1016/j.sandf.2018.03.002

Please cite this article in press as: Huang, M. et al., Bifurcation prediction of shear banding in sand with non-coaxial critical state model considering inherent anisotropy, Soils Found. (2018), https://doi.org/10.1016/j.sandf.2018.03.002

their non-coaxial terms are both developed on the 2D stress space. In view of the influence of the third stress invariant, the non-coaxial plasticity model is extended to the general 3D stress space by Qian et al. (2008) and applied to simulate biaxial tests, thereby obtaining better predicted results than those on the 2D stress space. Furthermore, this three-dimensional non-coaxial model is utilized to predict the experimental results of true triaxial tests by Huang et al. (2010). In the aforementioned works, it is noteworthy that the non-coaxial terms do not affect the stress-strain behavior described by the basic coaxial model in the proportional loading condition, and only facilitate the inception of shear banding. Thus, before performing a bifurcation analysis, a proper coaxial plasticity model should firstly be chosen that can describe the stress-strain-strength behavior of sand before strain localization.

Considering that the sand anisotropy induced by the gravitational deposition, which is inherently related to the micro-structural arrangement and affects the mechanical response of sand, is not characterized by the above-mentioned coaxial models, it is necessary for the anisotropic effect to be incorporated into the plasticity models for practical purposes. Hence, Pietruszczak and Mroz (2000) presented an anisotropic constitutive platform by assuming that the failure criterion depends on stress and the micro-structural tensors, termed fabric tensors. In the same way, Lade (2008) incorporated a micro-structural tensor into the Lade-Duncan failure criterion to develop a cross-anisotropic failure criterion. Based on the method suggested by Mortara (2009), Lu et al. (2011) formulated a cross-anisotropic failure criterion by modifying the elliptical shape function in the deviatoric plane, and predicted the onset of strain localization in cross-anisotropic soils. From the perspective of critical state soil mechanics (CSSM), another anisotropic constitutive platform is proposed on the basis of the dependence of the critical state line (CSL) in the void ratio $e$ versus mean effective stress $p$ space on sand-inherent fabric characteristics in relation to the loading direction. It assumes that the location of the CSL is not unique, but a proper function of scale-valued anisotropic state variable $A$. In the original works of Li and Dafalias (2002) and Dafalias et al. (2004), $A$ is defined as the first joint isotropic invariant of the fabric tensor proposed by Oda and Nakayama (1988) and a properly defined loading direction tensor. As a result, a function of the anisotropic critical state line (ACSL) is derived from that of the CSL proposed by Li and Wang (1988) by incorporating $A$. It should be emphasized that the concept of the ACSL is just an assumption which is made to fit into the framework of the CSSM on the basis of the experimental observations (Vaid and Chern, 1985; Vaid and Sivathayalan, 1996; Riemer and Seed, 1997; Yoshimine et al., 1998) that the critical state strength and the soil dilatancy are associated with the angle $\alpha$ of the major principal stress $\sigma_1$ in relation to the direction of the sand deposition, as illustrated in Fig. 1, and the intermediate principal stress ratio $b = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$. Although Yang et al. (2008) conducted a series of laboratory tests to confirm and calibrate the ACSL, the results are not accurate enough to obtain a determinate function form of the ACSL due to the scatter of the test data. Instead, Li and Dafalias (2012) started to debate once again that the CSL is unique on account of the motivation from the micromechanical and numerical experimental studies based on DEM. Therefore, there is still an ongoing discussion on the Unicity of the CSL and the proper form for the ACSL function. Moreover, the use of the anisotropic critical state model to conduct bifurcation analyses has not been addressed, especially for the case with various $\alpha$.

The aim of this paper is to make an attempt to predict the onset of strain localization according to the anisotropic constitutive platform proposed by Li and Dafalias (2002). In order to obtain a better simulation of the stress-strain-strength behavior of sand under a drained condition, some aspects of the model are revised to account for the anisotropic effect, involving the definition of an anisotropic state variable, the function form for the ACSL, the plastic modulus and the dilatancy function. Then, a corresponding non-coaxial model is developed based on the approach by Qian et al. (2008) to conduct bifurcation analyses. It can be seen that the predictions based on the non-coaxial model compare more favorably with the test results than those based on the coaxial model.

## 2. Effects of inherent fabric anisotropy

In the plasticity platform model developed by Li and Dafalias (2002), a symmetric second-order tensor-valued function proposed by Tobita (1989, 1988) is introduced to represent the interaction between the stress state and the internal fabric. The expression is given by

$$\widehat{T}_{ij} = \frac{1}{6}\left(\widehat{\sigma}_{ik}F^{-1}_{kj} + F^{-1}_{ik}\widehat{\sigma}_{kj}\right) = \widehat{p}\left(\widehat{r}_{ij} + \delta_{ij}\right) \tag{1}$$

where $F^{-1}_{ij}$ is the inverse of symmetric fabric tensor $F_{ij}$ related to vector magnitude $\Delta$ Curray, 1956 which describes both the orientation and the intensity of the inherent material anisotropy (Li and Dafalias, 2002; Oda et al., 1988). $\widehat{\sigma}_{ij}$ is a normalized stress which characterizes the loading direction. $\widehat{p} = \widehat{\sigma}_{ii}/3$ is the mean normal stress and $\widehat{s}_{ij}$ and $\widehat{s}_{ij} = \widehat{\sigma}_{ij} - \widehat{p}\,\delta_{ij}$ are the deviatoric parts. Tensor

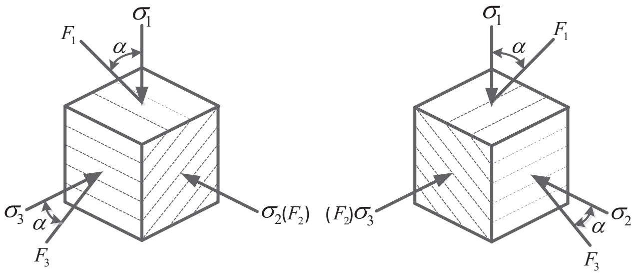

Fig. 1. Geometry interpretation of angle $\alpha$.

Please cite this article in press as: Huang, M. et al., Bifurcation prediction of shear banding in sand with non-coaxial critical state model considering inherent anisotropy, Soils Found. (2018), https://doi.org/10.1016/j.sandf.2018.03.002