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Effective Young's modulus of a spatially variable soil mass under a footing

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ABSTRACT

This study investigates the possibility of representing the effective Young's modulus (E_{eff}) for a footing problem supported on a *spatially variable* medium - the Young's modulus actually "felt" by the footing - using a spatial average. The E_{eff} is simulated by a homogenization procedure that matches the responses between a random finite element analysis (RFEA) and a homogeneous finite element analysis. Emphasis is placed on whether the spatial average can well represent the *numerical value* of E_{eff} in each spatially varying realization, not just the statistics of E_{eff} within an ensemble (a weaker requirement). It is found that the conventional spatial averaging model that treats all soil regions equally important cannot satisfactorily represent E_{eff}. Extensive numerical results show that the concept of "mobilization" is essential: highly mobilized soil regions close to the footing should be given larger weights than non-mobilized remote regions. Moreover, the non-uniform weights can be prescribed prior to RFEA, that is, they do not depend on the specific response corresponding to a specific random field realization. The "prescribed mobilization" for the spatially variable Young's modulus can be contrasted with the "emergent" mobilized shear strength in a spatially variable medium that results from the emergent nature of the critical failure path – it cannot be predicted prior to random finite element analysis. A key contribution of this paper is the development of a simple method based on the "pseudo incremental energy" to estimate the non-uniform weights for the spatial averaging using a single run of a homogeneous finite element analysis.

1. Introduction

The spatial variability of soil parameters has profound impact to the behavior of a geotechnical structure. In the literature, the impact of a spatially variable soil Young's modulus (E) on footing settlements has been widely studied [23,21,10,11,17,14,20,25,1,2,3]. For footings on soils with isotropic scales of fluctuation (SOF), an important observation made in Fenton and Griffiths [10,11] is that the statistics (mean and variance) for the "effective" Young's modulus (E_{eff}) are similar to those for the geometric average (E_g) over a prescribed domain under the footing. It is important to emphasize that E_{eff} is determined from the deformation response of a random finite element analysis (RFEA) (i.e., it is an output of a boundary value problem such as a footing applying pressure on top of a semi-infinite soil domain). The various spatial averages can be calculated from the input random field describing the spatial distribution of the Young's modulus over the semi-infinite soil domain (i.e., they are inputs unrelated to the boundary value problem).

It was reported in Fenton and Griffiths [10,11] that the settlement of a footing overlying a random field and a homogeneous field is the "same" in terms of second-moment statistics (mean and variance) if the homogeneous field is described by a suitable spatial average. Based on RFEA, Ching et al. [5,7] obtained a stronger conclusion for a soil square/cube subjected to displacement-controlled compression. They found that not only the statistics of E_{eff} can be well represented by a suitable spatial average but also E_{eff} is very strongly correlated to the spatial average. Note that the similarity in the statistics does not imply a very strong correlation, e.g., two random variables can have same statistics and yet be completely uncorrelated. This is a subtle but important point – a strong correlation between two random entities implies approximate agreement at the *realization* level, which is critical for reliability analysis, while comparable statistics merely imply agreement at the ensemble level. The latter is a weaker condition in the sense that the former implies that latter but not vice versa. Moreover, Ching et al. [7] found that this very strong correlation only exists for the soil square/cube subjected to displacement-controlled compression, not for the footing problem. For the footing problem, there is a significant scatter between E_{eff} and the spatial average, even though the statistics of E_{eff} can be well represented by the spatial average.

The purpose of this study is to propose a new spatial averaging method for the footing problem so that not only E_{eff} and the spatial

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average have similar statistics but they are also very strongly correlated, i.e., a new spatial averaging method that can effectively predict the "numerical value" of E_{eff} in *each realization*, not only its statistics at the ensemble level. It will be clear that the resulting spatial average is not a uniform "mobilization" but a non-uniform mobilization. The soil elements significantly influenced by the footing load are highly mobilized, whereas those remote to the footing have negligible mobilization. More importantly, it is found that the degree of mobilization can be well quantified by a certain quantity that is derived from the stress/strain change due to the footing load, and the spatial distribution of such a quantity can be obtained by a single run of a deterministic finite element analysis (FEA). The latter point is of critical practical importance. With the new spatial averaging method, it will be possible to simplify a RFEA involving a random field to a random variable problem which is less costly and perhaps more importantly, make probabilistic design more accessible to engineers. A single deterministic FEA is needed as a pre-processing step to compose the new spatial average (which is a random variable), but the cost of a single run is negligible compared to thousands of runs in a RFEA.

An important conclusion of this study is that the effective Young's modulus can be well represented by the spatial averaging with a *prescribed* non-uniform mobilization, which means that the non-uniform mobilization can be determined apart from the random field realizations and prior to RFEA. This is in contrast with the observations for shear strength: the authors [4,18,6,8] have shown that the mobilized shear strength is not the average along a prescribed curve but the average along the critical slip curve, which is an emergent curve that changes from realization to realization and cannot be predicted prior to RFEA. The possible reason for why this is true will be discussed in this paper.

2. Footing problem under investigation

2.1. Random field model

Consider a footing on a two-dimensional (2D) spatially variable soil mass, modeled by finite elements (FE) as shown in Fig. 1. The size of the footing is B = 2 m, and the soil mass has horizontal dimension = L = 20 m and depth = D = 10 m. The spatially variable Young's modulus, denoted by E(x,z), is modeled as a stationary lognormal random field with inherent mean = μ and inherent coefficient of variation (COV) = V. To define the correlation structure between two locations with horizontal interval distance = Δx and vertical interval distance = Δz, the single exponential auto-correlation model is considered [27–28]:

$$\rho(\Delta x,\Delta z) = \exp(-2|\Delta x|/\delta_x - 2|\Delta z|/\delta_z) \quad (1)$$

where δ_x and δ_z are the SOFs in the (x,z) directions, respectively, for the

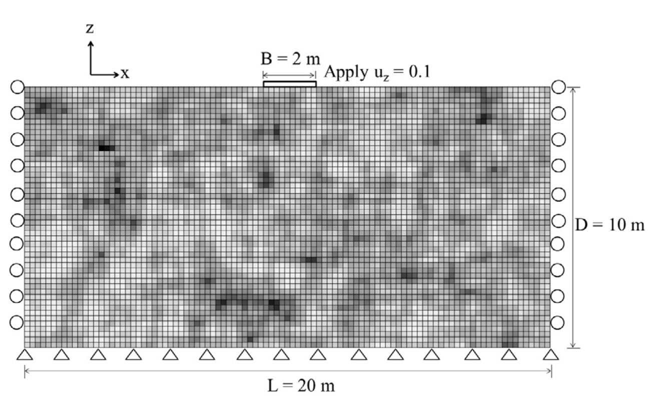

Fig. 1. Realization of the E random field for the 2D footing problem with $\delta_x = \delta_z = 1$ m.

ln[E(x,z)] random field. The ln[E(x,z)] random field is a 2D stationary normal random field with mean = $\lambda = \ln[\mu/(1+V^2)^{0.5}]$ and variance = $\xi^2 = \ln(1+V^2)$. In this study, the local average for ln(E) of each FE is taken to be the arithmetic average for ln[E(x,z)] over the element, and this local average is simulated using the Fourier series method [19,9]. The E of each FE is simulated as the exponential of its local average for ln(E). The E of each FE is in fact the geometric average for E(x,z) over the element. The mean value of this E is not exactly μ [13]. The correction factor $(1+V^2)^{0.5\times(1-\Gamma^2)}$ is multiplied to the simulated E to ensure the mean value is μ, where Γ^2 is the variance reduction factor for the local averaging effect (see Eq. 22 in [13] for the correction factor and Eq. 15 for the definition of Γ^2). In the case that the sizes of the element are significantly smaller than the SOFs, a geometric average is roughly the same as an arithmetic average (correction factor ≈ 1). Fig. 1 shows a realization of the E random field with $\delta_x = \delta_z = 1$ m. The E value for the light region is low, while that for the dark region is high. The Poisson's ratio (ν) is assumed to be constant (ν = 0.3), because the impact of the spatial variability of the Poisson's ratio is insignificant [10,11,5].

2.2. Finite element model

The 20 m × 10 m plane strain rectangular domain is modeled by the FE mesh shown in Fig. 1. Each FE is a 4-noded element of size = 0.2 m × 0.2 m. In total, there are 100 × 50 = 5000 elements with reduced integration (CPE4R). Each FE follows an isotropic elasticity model with E = its *local* geometric average multiplied by the correction factor $(1+V^2)^{0.5\times(1-\Gamma^2)}$, ν = 0.3, and unit weight γ = 20 kN/m^3. The nodes along the vertical boundary are constrained against horizontal displacement (roller, see Fig. 1), whereas the nodes on the bottom boundary are fixed (hinge). The top boundary is free, except the B = 2 m line segment under the footing. The footing is assumed to be rigid and the soil-footing interface is assumed to be rough. The Young's modulus of the soil mass is modeled as a stationary lognormal random field with inherent mean = μ = 20,000 kN/m^2 and inherent coefficient of variation V = 1.0. Cases with $\delta_x = \delta_z = \delta$ will be first considered. Five SOFs are considered: δ = 1 m, 2 m, 5 m, 10 m, 100 m, and 1000 m (δ/B = 0.5, 1, 2.5, 5, 50, and 500). For each δ, one thousand realizations of E random fields are simulated.

2.3. Simulation of effective Young's moduli

For each random field realization, a geostatic step is adopted to build up the in-situ stress field over the entire soil mass. Then, the footing is loaded with a vertical downward uniform displacement u_z = 0.1 m in the FE simulation, not allowing any rotations. This is an important practical case – footings cannot rotate because they are constrained by ground beams. Footings can rotate in other cases, e.g. a monopile supporting a wind turbine. This (a footing that can rotate) will be addressed in our future work. The resulting total contact force between the footing and the soil mass is recorded. Another FE simulation with homogeneous E is conducted, following the same geostatic step and the same displacement-controlled loading. The homogeneous E value is adjusted until the total contact force matches that for the random field realization. The adjusted E value is called the effective Young's modulus, E_{eff}, for the random field realization. This process is sometimes called "homogenization" [16,15,22]. It is worth emphasizing E_{eff} is a response (or an output) from a RFEA.

2.4. Results for cases with $\delta_x = \delta_z$

Fig. 2 shows the pairwise plot for the simulated E_{eff}/μ versus (spatial average)/μ for cases with $\delta_x = \delta_z = \delta$, where the spatial average is the geometric average (E_g) over the 1B × 5B domain under the footing. This averaging domain was considered in Fenton and Griffiths [10]. This spatial averaging model will be denoted by the E_g model. Although