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# Ultimate Strength Analysis of Laminated Composite Sandwich Plates

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ABSTRACT

The ultimate strength analysis of laminated composite sandwich plates is presented via ultimate failure load prediction along with modes of failure for laminated composite sandwich plates using mathematical model based on improved higher order shear deformation theory (IHSDT). The proposed IHSDT mathematical model satisfies the inter-laminar shear stress continuity at each layer interface and also ensures zero transverse shear stress conditions at the plate top and bottom. The piecewise parabolic shear stress variation across the thickness of each layer is considered. No shear correction factors are required. The suitable $C_0$ FE formulation of IHSDT model is developed by authors.

## 1. Introduction

Composites, in the present era, are emerging as a replacement of conventional materials used for the structure. Its ever-growing demand has led to many types of research over the material. The strength study is one of the most important aspects of laminated composite plates. Many theories have been employed in its study. The strength has been tried to be accessed using different theories. The first comprehensive theory developed for plates was Kirchhoff's classic laminate plate theory (CPT) [1] but the theory was unable to predict accurate results for thick plates and the reason was that the theory does not take into consideration of the transverse shear strains. The first order shear deformation theory (FSDT) [2,3] theories predicted the reasonable good results for thin and thick plates which accounted for transverse shear effects but it involved shear correction factor to maintain zero traction at top and bottom plate surfaces. These correction factors were geometry, boundary and loading conditions dependent. To remove this complex factor higher shear deformation theory (HSDT) came into existence which was free from shear correction factors. But still the transverse shear stress continuity conditions at the interfaces between layers remained infringed. So various improved form of higher-order theories are proposed by researchers till date.

Based on the FSDT, Reddy and Pandey [4] introduced an FE method for the failure analysis of composite plates. Yang et al. [5] analyzed laminated anisotropic plate using first order shear deformation theory (FSDT). It can be said that the HSDT have been studied extensively by Levinson [6]. Reddy, as well as Murthy [6,7], have developed HSDT for analysis of laminated composites. Reddy [7] also introduced a mixed formulation for his theory later on. Engblom and Ochoa [8,9] with enhanced interpolation in the thickness direction carried a two-dimensional plate analysis.

Many researches have been done for first ply failure. But composites as a whole fails at a much higher load than that of first ply failure load. This has led to, study of ultimate ply failure load which involved the progressive failure analysis of laminated plates.

Progressive failure analysis is a complex study as the failure develops gradually in every lamina for a laminate. The analysis of first ply failure only exhibits the weakest lamina in the laminate in which stress exceeds than that of allowable stress. But after first ply failure, the stiffness decreases for that particular element only while for other it remains unchanged, therefore the ply is again analyzed to check whether it can hold an extra amount of load till the ultimate strength is reached.

The various methods have been employed by researchers to carry out the progressive failure analysis using different theories and methods for calculation of ultimate strength of laminated composite structures. Reddy and Reddy [10] using layerwise theory and Tsai-Wu failure criteria introduced an algorithm for study of laminated composite plates for the nonlinear progressive failure, from which they concluded that in order to predict accurate failure loads a 3D analysis is required. The progressive failure analysis was also done by Liu et al. [11] using different failure criterion under low-velocity impact on carbon fibre laminated composite plates. The study provides a conclusion that appropriate selection and use of failure criteria may result in achieving high fidelity and efficiency in the analysis. Tian et al. [12] analyzed the composite plates for biaxial tension and compression for ultimate failure through improved micromechanical computation using FE method. Lee et al. [13] has employed the Puck failure criterion for

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**Notations**

| | |
|---|---|
| $u_\alpha{}^0$ | in-plane displacement |
| $nl$ and $nu$ | number of lower and upper layers |
| $T_\alpha{}^k$ and $S_\alpha{}^k$ | $k$th layer slopes subsequent to lower and upper layers respectively |
| $H(z-z_k{}^u)$ and $H(-z+z_k{}^l)$ | unit step functions |
| $\xi_\alpha$, $\phi_\alpha$ | higher order unknown terms |
| FL | failed location on the laminate (1 = bottom, 2 = middle, 3 = top) |
| FPL | failed-ply number within the laminate |
| FGP | failed Gaussian point number within the failed element |
| FEL | failed element number in the finite element mesh |
| M | transverse matrix cracking |
| F | fibre breakage |
| IFS | inter-fibre matrix shear failure |
| FPF | first ply failure |

progressive failure analysis of laminated composite plates using damage coupled FE method which confirmed that simulation study can be carried to predict the failure load rather than experimental study.

Sun et al. [14] used micromechanics theories with non-iterative element-failure method to analyze the progressive failure in laminated composite plates. Akhras and Li [15] used Cho's higher-order zigzag laminate theory for the progressive failure analysis but with the spline finite strip method and Lee's failure criterion. Prusty [16] studied the progressive failure of laminated stiffened/unstiffened composite plates under the influence of transverse static load. In the study, different failure theories were applied for the First Ply Failure analysis but only Tsai–Wu failure theory was used for the progressive failure analysis.

Ganesan and Zhang [17] had investigated the ultimate failure analysis of laminates with uniform thickness for uniaxial compression. Singh et al. [18–20] in their various researches had investigated the plates with uniform thickness for calculation of ultimate strength. Ahmed and Sluys [21] developed a computational model to present a failure model for delamination, matrix cracking and the collective effect. The model showed that using quadratic tensor theories accurate results for failure envelopes of 3D four-directional and five-directional braided composites can be predicted whereas maximum strain criteria can be used for prediction of 3D six-directional and seven-directional braided composites. Qilin and Zhen [22] analyzed composite plates for failure analysis using $C^0$ element and using Reddy's Theory.

Turan et al. [23,24] analyzed carbon epoxy laminated composited plates for failure loads and modes of ultimate failure as well as compared to that of experimental results. Aktas and Karakuzu [25] analyzed carbon-epoxy composite plate for failure strength and failure mode of arbitrary orientation.

The gap findings from the literature are, that there are no studies predicting failure mode of laminated composite sandwich plates and no results on failure load of rectangular laminated composite sandwich plates using robust model of IHSDT. Further, there is no work in literature enlightening each ply by ply failure load with its location along with the mode of failure of laminated composite sandwich plates.

Based on the above findings in the present paper, a 2D mathematical model based on IHSDT is developed and its $C_0$ finite element implementation is coded in FORTRAN for the progressive failure analysis of laminated composite sandwich plates. The transverse shear stress continuity at each layer is ensured which is important for modeling sandwich plates. It also full-fill the condition of zero shear stresses at end surfaces of the laminated plates. The consideration of piecewise parabolic shear stress variation across the thickness of each layer makes the model efficient for the sandwich plate. Thus, present IHSDT model is equipped with all features required for efficient shear modeling of laminated composite sandwich plates. The present model can predict failure mode, nondimensional failure load and the location of failure for each ply in laminated composite sandwich plates. To overcome the problem of the $C_1$ continuity related with the IHSDT model, a suitable a $C_0$ FE formulation has been developed. A nine nodded elements, with seven unknowns at each node is used in present FE formulation.

The different failure criterions considered in the present analysis are: Tsai-Wu's, Maximum stress, Hoffman's and Tsai-Hill. The comparison of the proposed 2D model is done for deflection and stresses with Pagano's [26] 3D exact solutions. The accuracy of the proposed 2D FE model is also checked for FPF load with Cho and Yoon [27] 3D exact solution and based on first order shear deformation theory FE solution by Reddy and Reddy [28]. The ultimate failure load of plies is also compared with the results of Akhras et al. [15]. The values of central deflection in laminated composite plate are compared graphically with the results of Pervez et al. [29], Reddy [30] and Noor and Burton [31]. The mean deflections of laminated sandwich plates are compared with the 3D elasticity solution of Pagano [26], HSDT1 (Higher order shear deformation theory with the free shear stress conditions on the top and bottom bounding planes imposed) and HSDT2 (Higher order shear deformation theory without the free shear stress conditions on the top and bottom bounding planes imposed) of Pandya and Kant [32], and closed form and FEM solution of Reddy and Chao [33]. All the comparison results are in good agreement with present model results. Many new numerical examples of angle-ply and cross-ply laminates with different thickness, lamination schemes and different complex boundary conditions are investigated which may be useful for designers.

## 2. Mathematical model

For the present analysis, the following equation of In-plane displacement fields is being adopted and is also shown in Fig. 1.

$$u_\alpha = u_\alpha^0 + \sum_{k=0}^{nu-1} S_\alpha^k (z - z_k^u) H(z - z_k^u) + \sum_{k=0}^{nl-1} T_\alpha^k (z - z_k^l) H(-z + z_k^l) + \xi_\alpha z^2 + \phi_\alpha z^3 \quad (1)$$

subscript $\alpha$ shows the coordinate directions. [$\alpha$ *being* 1, 2 i.e. $x$, $y$ in this case].

For constant transverse displacement over the plate thickness, an assumption is considered i.e.,

$$u_3 = w(x,y) \quad (2)$$

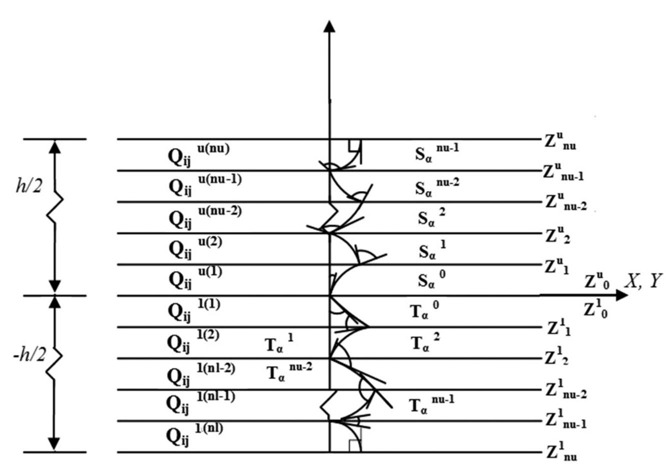


**Fig. 1.** General lamination layup and in-plane displacement across the cross-section of a plate.