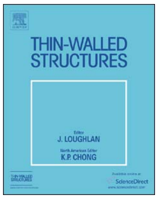
Thin-Walled Structures 125 (2018) 281–293

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws



Nonlinear primary resonance of functionally graded porous cylindrical shells using the method of multiple scales

Kang Gao*, Wei Gao, Binhua Wu, Di Wu, Chongmin Song

School of Civil and Environmental Engineering, The University of New South Wales, Sydney NSW 2052, Australia

ARTICLE INFO

Keywords:
FG porous cylindrical shell
Nonlinear primary resonance
Free vibration
Nonlinear forced vibration
Multiple scales method

ABSTRACT

An analytical method is proposed for the nonlinear primary resonance analysis of cylindrical shells made of functionally graded (FG) porous materials subjected to a uniformly distributed harmonic load including the damping effect. The Young's modulus, shear modulus and density of porous materials are assumed to vary through the thickness direction based on the assumption of a common mechanical feature of the open-cell foam. Three types of FG porous distributions, namely symmetric porosity distribution, non-symmetric porosity stiff or soft distribution and uniform porosity distribution are considered in this paper. Theoretical formulations are derived based on Donnell shell theory (DST) and accounting for von-Kármán strain-displacement relation and damping effect. The first mode of deflection function that satisfies the boundary conditions is introduced into this nonlinear governing partial differential equation and then a Galerkin-based procedure is utilized to obtain a Duffing-type nonlinear ordinary differential equation with a cubic nonlinear term. Finally, the governing equation is solved analytically by conducting the method of multiple scales (MMS) which results in frequency-response curves of FG porous cylindrical shells in the presence of damping effect. The detailed parametric studies on porosity distribution, porosity coefficient, damping ratio, amplitude and frequency of the external harmonic excitation, aspect ratio and thickness ratio, shown that the distribution type of FG porous cylindrical shells significantly affects primary resonance behavior and the response presents a hardening-type nonlinearity, which provides a useful help for the design and optimize of FG porous shell-type devices working under external harmonic excitation.

1. Introduction

An amazingly creative invention named functionally graded materials (FGMs) was proposed by material scientists during the spacecraft project in 1984, as a means of ultrahigh temperature resisting materials. Since then, the studies about FGM have been completely blooming in almost all associated fields. Generally, FGMs are made from a mixture of metallic and ceramic ingredients. However, a novel functionally graded (FG) pure metallic porous material was proposed in recent years by changing the cell geometry, density and/or material composition from point to point within the porous foams or metallic foams in the process of fabrication [1–4]. Such materials can be widely used in various industries such as energy absorbing systems, porous electrodes, sound absorbers, heat exchangers, construction materials, electromagnetic shielding, etc., due to excellent impact energy absorption, high specific strength, and low thermal conductivity and other special characteristics [5–8]. Dynamic characteristics of FG porous structures keep attracting research attentions as the application of such structures is becoming diversification and more complex.

As one of the most promising materials in lightweight structures, the mechanics and mechanism of FG porous structures have been investigated extensively in the past. Nearly all of the early researchers regarding FG porous structures were focused on problems of elastic buckling or dynamic buckling analysis. Magnucki and his co-workers firstly and thoroughly investigated the static and dynamic stability of FG porous structures (such as porous beams [9] or porous sandwich beam [10], porous plates [11,12] or porous sandwich plates [13] and porous circular cylindrical shells [14–17]) based on theoretical and finite element methods. For example, Magnucki and Stasiewicz [9] studied the buckling behavior of a symmetric porosity distributed bar based on the principle of stationarity of the total potential energy including the effect of shear strain. The buckling and strength of a sandwich beam with a metal foam core were presented by Magnucka-Blandzi and Magnucki [10]. Magnucka-Blandzi [11,13] also investigated the dynamic buckling of a porous circular plate and static buckling of a rectangular sandwich plate with the porous core. Furthermore, Belica and Magnucki [14–17] employed a nonlinear hypothesis of deformation of a plane cross-section and Hamilton's

---

* Corresponding author.
E-mail address: kang.gao@unsw.edu.au (K. Gao).

https://doi.org/10.1016/j.tws.2017.12.039
Received 29 September 2017; Received in revised form 21 November 2017; Accepted 27 December 2017

principle to carry on analytical and numerical studies on dynamic buckling of porous cylindrical shells subjected to external pressure and axial compression. Porous structures are not only subjected to different loading conditions but also saturated with liquid and gas. Under such circumstances, the dynamic properties or mechanical mechanism would be different. Therefore, Jabbari et al. [18–22] studied the stability of saturated FG porous circular plate with or without piezoelectric layers subjected to a radially loading, thermal buckling and combined thermal and mechanical loads on the ground of linear poroelasticity theory of Biot [23]. Following their work, the post-buckling behavior of saturated FG porous circular plates subjected to a uniformly radially loading with simply-supported and clamped boundary conditions was presented by Feyzi and Khorshidvand [24].

Natural frequencies and other nonlinear dynamic properties play an important role in the design [25–27] and analysis of FG porous structures in the large amplitude deflections. More and more researches paid attention to the dynamic behavior of FG porous structures in the engineering practices. Chen et al. [28] applied the Timoshenko beam theory and Lagrange equation to investigate the free and forced vibration characteristics of shear deformable FG porous beams with symmetric and asymmetric porosity distributions. Furthermore, by using the same method, the nonlinear free vibration of shear deformable sandwich beams with an FG porous core was studied [29]. In this study, they also considered a non-symmetric porosity distribution and a uniform porosity distribution when compared with Magnucki [10] and the results showed that with the consideration of two layers at the top and bottom sides, the vibration behavior of the structure is improved. The rapid development of manufacturing technique makes it possible to introduce nanofillers such as carbon nanotubes (CNTs) and graphene platelets (GPLs) into porous materials. The novel porous nanocomposites occupy both advantages of CNTs or GPLs and porous materials. Kitipornchai et al. [30] presented the influence of both porosity and GPLs dispersion pattern on the free vibration of FG porous nanocomposite beams. Following this idea, Chen et al. [31] studied the nonlinear vibration and postbuckling behavior of GPLs reinforced FG porous beams. Based on the Reddy's third-order shear deformation theory, the dynamic characteristics of a porous rectangular plate resting on a Pasternak foundation was solved by differential quadrature method [32]. Ebrahimi and Habibi [33] employed the third order shear deformation plate theory and finite element method to predict the deflection and vibration characteristics of a saturated FG porous rectangular plate. There are also some studies dealing with the random distribution of porosity during the multi-step sequential infiltration technique. Wattanasakulpong and Ungbhakorn [34] conducted an investigation of random porosity volume fraction on the linear and nonlinear vibration of FG porous beams with elastically restrained ends. Using the semi-analytical differential transform method, the free vibration of rotating FG porous beam was studied by Ebrahimi and Mokhtari [35] based on the Timoshenko beam theory. They [36] also presented the free vibration of a rotating double-tapered functionally graded (FG) porous beam based on Euler–Bernoulli beam theory and then the governing equation is solved by the differential transform method.

As can be seen, most of those works about FG porous structures are focus on beam and plate structures, and investigations involving the application of FG porous cylindrical shells are still limited in number. However, in engineering practices, the cylindrical shell/sheet structures are widely used in all kinds of fields in order to match the desired functionality and optimize the structures [37–39], such as the propellant tank of the space shuttle, the skin of the ballistic missile, oil refineries, petrochemical plants, power plants, pressure vessel and so on. Ghadiri and SafarPour [40] applied the first-order shear model and modified couple stress theory to analyze the free vibration characteristics of FG porous microshell in the thermal environment. Wang and Wu [41] calculated the natural frequencies of an FG porous cylindrical shell different boundary conditions using sinusoidal shear deformation theory and Rayleigh-Ritz method. Additionally, the understanding of free vibration and nonlinear vibration analysis is crucial to FG porous cylindrical shells, however, no previous work has been done for FG porous cylindrical shells with external harmonic excitation, especially for the resonant characteristics with different internal porosity distributions. Thus, it is of a great importance to analyze the forced vibration behavior of FG porous cylindrical shell due to the time-dependent external forces and the proper understanding and development of primary resonance of FG porous cylindrical shell can help engineers avoid the peak resonances of the structural system in the design process.

The purpose of this paper is to study the nonlinear forced vibration characteristics of the cylindrical shell made of functionally graded porous materials subjected to a uniformly distributed harmonic loading. The nonlinear compatibility equation is derived by using the Donnell shell theory with the consideration of von-Kármán strain-displacement relation and damping effect. With an acceptable accuracy, neglecting the inertia and rotary inertia terms, the single-mode approximation of deflection was assumed, which satisfies the boundary conditions [42,43]. Then the Galerkin method in conjunction with the method of multiple scales is used to obtain a second-order nonlinear ordinary equation with the cubic nonlinear term, named Duffing-type equation. Based on this equation, frequency-response analysis is investigated for three types of FG porous cylindrical shells, that are symmetric porosity distribution, non-symmetric porosity stiff or soft distribution, and uniform porosity distribution. The influences of porosity distribution, porosity coefficient, damping ratio, amplitude and frequency of the external harmonic excitation, aspect ratio and thickness ratio on the nonlinear dynamic behavior are discussed in details.

2. Material gradient of an FG porous cylindrical shell

In this paper, three types of FG porous distributions, namely Type 1(symmetric porosity distribution) [9–16], Type 2 (non-symmetric porosity distribution) [19–22,41] and Type 3 (uniform porosity distribution) are considered in cylindrical shells, as shown in Fig. 1. The elastic modulus and mass density of porous materials vary through the thickness direction based on the assumption of a typical mechanical feature of the open-cell metal foam. The variation of Young's modulus $E(z)$, shear modulus $G(z)$ and density $\rho(z)$ through the thickness direction of the cylindrical shell is described by Eqs. (1)–(4).

A novel non-symmetric porosity soft distribution is also proposed and Young's module E, shear modulus G and density ρ of this type of distribution gradually become small from inside diameter to outside diameter, as shown in Fig. 1(c). Though non-symmetric porosity soft distribution is different from that of stiff type, both of them occupy similar free vibration and forced vibration behavior due to the equal stiffness and mass. Thus, for the convenience, in this paper, both non-symmetric porosity stiff distribution and soft distribution are called Type 2.

Type1: symmetric porosity distribution

$$\begin{aligned} E(z) &= E_{\max}\left[1 - N_0 \cos\left(\frac{\pi z}{h}\right)\right] \\ G(z) &= G_{\max}\left[1 - N_0 \cos\left(\frac{\pi z}{h}\right)\right] \\ \rho(z) &= \rho_{\max}\left[1 - N_m \cos\left(\frac{\pi z}{h}\right)\right] \end{aligned} \tag{1}$$

Type 2: non-symmetric porosity stiff distribution