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Finite element analysis of vibrating micro-beams and -plates using a three-dimensional micropolar element

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ABSTRACT

The micropolar theory (MPT), through taking the rotational degrees of freedom of material particles into account, is a suitable elasticity theory for the mechanical analysis of microstructures. In this article, the vibration behavior of microscale beams and plates is studied based on MPT. To this end, first, a three-dimensional (3D) formulation is developed for the micropolar continua which can be readily used in the finite element analyses. Then, a non-classical 3D element is introduced to investigate the free vibration characteristics of micropolar beams and plates. The microstructure effect on the frequencies of microbeams and microplates under different kinds of boundary conditions is illustrated. Also, the results of MPT are compared with those of classical theory and it is indicated that there is a considerable difference between their predictions at small scales.

1. Introduction

It is experimentally revealed that the mechanical behavior of structures at small scales becomes size-dependent [1–6]. Therefore, although the classical continuum mechanics is regarded as an efficient approach for the analysis of large scale structures, its use in the problems of micro- and nano-structures is in doubt. Up to now, a variety of non-classical continuum theories have been developed with the aim of capturing the small scale effect on the mechanical behavior of micro- and nano-structures. The strain gradient [7–9], nonlocal [10,11], couple stress [12–14], micropolar [15,16], micromorphic [17] and surface stress [18,19] elasticity theories are popular non-classical continuum approaches whose applications to the problems of small-scale structures have been reported in many research works [20–29].

The concept of micropolar or Cosserat elasticity theory was firstly proposed by Cosserat brothers in 1909 [30]. Later, Eringen and Suhubi [15,16], Günther [31], Ieşan [32], Nowacki [33] and some other researchers have developed the present known micropolar theory (MPT). MPT is a size-dependent field theory including the microstructure effect. Based on this theory, at each material point of the continuum, a microstructure is considered which is rigid and can rotate independently from the neighbouring medium [34]. Accordingly, each material particle of micropolar continua has six degrees of freedom (DOF) including three translational and three rotational ones. This is while, nine extra DOFs consists of micro-rotations, micro-stretches and micro-shears are considered in the micromorphic theory (MMT) [35–39]. In fact, contrary to the assumption of rigid deformations in MPT, micro-motions of MMT are described by three deformable vectors, i.e. directors. MPT can be used for both fluids [40] and elastic solids [41]. A literature survey shows that this theory has been used for different materials such as granular materials [42], heterogeneous materials [43–45] and biomaterials [46,47] as well as in the context of size-dependent analysis of small-scaled structures [48–51].

There are several papers on the mechanical behavior of structures using MPT. Herein, some of them are cited. Anderson [52] studied the forced vibration characteristics of elastic bodies based on MPT. Using the finite element method (FEM), the bending behavior of micropolar elastic plates was investigated in [53,54]. Zhang et al. [55] analyzed the multi-body contact of micropolar materials. Pompei and Rigano [56] addressed the bending problem of micropolar viscoelastic plates. Sargsyan and Sargsyan [50] studied the dynamic bending of isotropic micropolar elastic thin plates with independent fields of displacements and rotations.

Since the governing equations of non-classical continuum models are mathematically complex, conducting analytical solution approaches for them may be impossible or difficult in many cases (e.g. see [57,58]). Hence, introducing numerical tools in this field can be of great importance. Among different numerical techniques, FEM has the potential to be efficiently used in the field of micro- and nano-mechanics

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[59–66]. However, the conventional elements of FEM are not suitable for predicting the size-dependent behavior of materials as they are based on the classical continuum mechanics. In this respect, some attempts have been made to develop size-dependent elements including the effects of micro- and nano-scale. Recently, Ansari et al. [67] proposed a non-classical Timoshenko beam element within the framework of Mindlin's strain gradient theory. In comparison with the classical Timoshenko beam element, their developed element needs two additional nodal DOFs including derivatives of lateral translation and rotation, which means a total of 4 DOFs per node. It was indicated that the new element is able to predict the bending and vibration responses of microbeams under different types of end conditions with considering strain gradient effects.

In the current paper, based upon MPT, a non-classical element is developed to investigate the free vibration behavior of microbeams and microplates with various kinds of boundary conditions. The proposed element can capture the microstructure effect through considering the micro-rotation of material particles. Also, it is three-dimensional and can be applied to different micropolar elastic bodies. In order to develop the element, the micropolar elasticity is formulated in a new and general way. The matrix representation of relations is given which is appropriate for the finite element modeling. Selected numerical results are presented to study the microstructure effect on the vibration behavior of microbeams and microplates.

2. Formulation of micropolar theory

If $\tilde{\boldsymbol{\sigma}}$ and $\tilde{\boldsymbol{\mu}}$ denote the stress and couple stress tensors, respectively, the governing equations of motion of a micropolar continuum are formulated as [41,68]

$$\mathrm{div}(\tilde{\boldsymbol{\sigma}}) + \rho\mathbf{f} = \rho\ddot{\mathbf{u}},\ \mathrm{div}(\tilde{\boldsymbol{\mu}}) + \rho\mathbf{m} - \boldsymbol{\epsilon} : \boldsymbol{\sigma} = \rho j\ddot{\boldsymbol{\phi}} \tag{1}$$

whose indicial representations in the context of Cartesian coordinate system are given by

$$\sigma_{ij,j} + \rho f_i = \rho\ddot{u}_i, \quad \mu_{ij,j} + \rho m_i - \epsilon_{ijk}\sigma_{jk} = \rho j\ddot{\phi}_i \tag{2}$$

in which $\mathbf{f}$ and $\mathbf{m}$ respectively stand for the body force and body couple; $\mathbf{u}$ and $\boldsymbol{\phi}$ are the displacement and micro-rotation vectors and $\boldsymbol{\epsilon}$ shows the permutation symbol. Also, ρ and j denote the mass density and micro-inertia, respectively. As it was expressed earlier, the rigid micro-deformation vector $\boldsymbol{\phi}$ is the simplified director in the micropolar elasticity theory.

The following linear micro-strain tensors are defined [16,48]

$$\begin{aligned} \tilde{\boldsymbol{\varepsilon}} &= \mathrm{Grad}(\mathbf{u}) + \boldsymbol{\epsilon}\boldsymbol{\phi}, \quad \varepsilon_{ij} = u_{i,j} + \epsilon_{ijk}\phi_k \\ \tilde{\boldsymbol{\eta}} &= \mathrm{Grad}(\boldsymbol{\phi}), \quad \eta_{ij} = \phi_{i,j} \end{aligned} \tag{3}$$

to write the strain energy density for a linear elastic solid as [68]

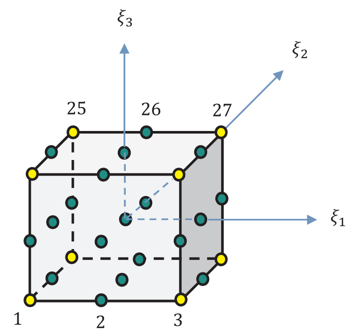
$$\begin{aligned} \hat{W} &= \frac{1}{2}(\lambda \mathrm{tr}^2(\tilde{\boldsymbol{\varepsilon}}) + (\mu + \kappa)\mathrm{tr}(\tilde{\boldsymbol{\varepsilon}}\tilde{\boldsymbol{\varepsilon}}^{\mathrm{T}}) + \mu \mathrm{tr}(\tilde{\boldsymbol{\varepsilon}}^2) + \alpha \mathrm{tr}^2(\tilde{\boldsymbol{\eta}}) + \beta \mathrm{tr}(\tilde{\boldsymbol{\eta}}^2) + \gamma \mathrm{tr}(\tilde{\boldsymbol{\eta}}\tilde{\boldsymbol{\eta}}^{\mathrm{T}})) \\ &= \frac{1}{2}(\lambda\varepsilon_{ii}\varepsilon_{jj} + (\mu + \kappa)\varepsilon_{ij}\varepsilon_{ij} + \mu\varepsilon_{ij}\varepsilon_{ji} + \alpha\eta_{ii}\eta_{jj} + \beta\eta_{ij}\eta_{ij} + \gamma\eta_{ij}\eta_{ji}) \end{aligned} \tag{4}$$

where the classical Lame coefficients are denoted by λ, μ; and α, β, γ, κ are some material constants of micropolar materials.

As a result, the stress and couple stress tensors are obtained as [15,16,68]

$$\tilde{\boldsymbol{\sigma}} = \frac{\partial \hat{W}}{\partial \tilde{\boldsymbol{\varepsilon}}}, \quad \tilde{\boldsymbol{\mu}} = \frac{\partial \hat{W}}{\partial \tilde{\boldsymbol{\eta}}} \tag{5}$$

Thus, one can have [68]



$$\boldsymbol{\xi} = [\xi_1 \quad \xi_2 \quad \xi_3], \qquad -1 \le \xi_i \le 1, \qquad i = 1,2,3$$

$$\mathbf{d}_j(\boldsymbol{\xi}) = [u_1(\boldsymbol{\xi}) \quad u_2(\boldsymbol{\xi}) \quad u_3(\boldsymbol{\xi}) \quad \phi_1(\boldsymbol{\xi}) \quad \phi_2(\boldsymbol{\xi}) \quad \phi_3(\boldsymbol{\xi})]^{\mathrm{T}}, \qquad j = 1, \ldots, 27$$

Fig. 1. Cubic micropolar element.

$$\tilde{\boldsymbol{\sigma}} = \lambda \mathrm{tr}(\tilde{\mathbf{e}})\mathbf{I} + 2\mu\tilde{\mathbf{e}} + \kappa\tilde{\boldsymbol{\varepsilon}}, \quad \tilde{\boldsymbol{\mu}} = \alpha \mathrm{tr}(\tilde{\boldsymbol{\eta}})\mathbf{I} + \beta\tilde{\boldsymbol{\eta}}^{\mathrm{T}} + \gamma\tilde{\boldsymbol{\eta}} \tag{6}$$

in which $\mathbf{I}$ is the identity tensor and $\tilde{\mathbf{e}}$ denotes the classical linear strain that is expressed as

$$\tilde{\mathbf{e}} = \frac{1}{2}(\mathrm{Grad}(\mathbf{u}) + \mathrm{Grad}^{\mathrm{T}}(\mathbf{u})) \tag{7}$$

and

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad 2\mu + \kappa = 2G = \frac{E}{(1+\nu)} \tag{8}$$

Detailed information about material constants can be found in Section 4.

Since micro-strain tensors of Eq. (3) are un-symmetric, the stress and couple stress tensors of Eq. (6) are un-symmetric too, which in the Cartesian coordinate system are given by

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} + \kappa\varepsilon_{ij}, \quad \mu_{ij} = \alpha\eta_{kk}\delta_{ij} + \beta\eta_{ji} + \gamma\eta_{ij} \tag{9}$$

Using the following definition [48]

$$e_{ij} = \frac{1}{2}(\varepsilon_{ij} + \varepsilon_{ji}) \tag{10}$$

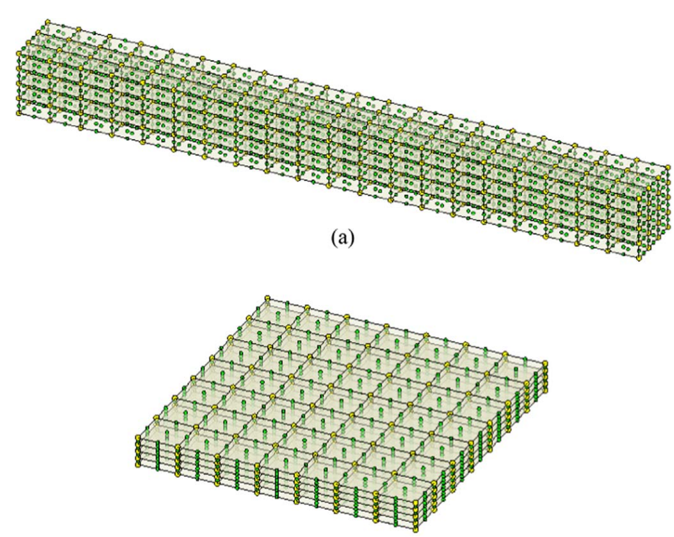


Fig. 2. Schematic view of discretized 3D micropolar (a) beam and (b) plate.