Full length article

Wave propagation in viscoelastic thin cylindrical nanoshell resting on a visco-Pasternak foundation based on nonlocal strain gradient theory

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ABSTRACT

Wave propagation in viscoelastic single walled carbon nanotubes is investigated by accounting for the simultaneous effects of the nonlocal constant and the material length scale parameter. To this end, thin shell theory is used to model the viscoelastic single walled carbon nanotubes, and the nonlocal strain gradient theory is used to account for the effects of the nonlocal constant and the material length scale parameter. The Kelvin–Voigt model is used to model the viscoelastic property, and the governing equations are derived through Hamilton's principle. The viscoelastic single walled carbon nanotube medium is modeled as visco-Pasternak. The results demonstrate that viscoelastic single walled carbon nanotube rigidity is higher in the strain gradient theory and lower in the nonlocal theory in comparison to that in the classical theory. Also, the size effects, nanotube radius, circumferential wavenumber, nanotube damping coefficient, and foundation damping coefficient exert considerable effects on viscoelastic single walled carbon nanotube phase velocity.

1. Introduction

Carbon nanotubes (CNTs) were discovered by Iijima [1] in 1991 and quickly attracted attention in various research areas, such as mechanics, electronics, and medicine. The distinctive mechanical properties of these single walled carbon nanotubes (SWCNTs), including high tensile strength and low density, led to their extensive engineering application in the field of mechanics. To make better use of these nanostructures, it is imperative to identify their mechanical and dynamic properties. Instruments used to predict the dynamic behavior of SWCNTs include the molecular dynamics (MD) method as well as experimental methods. However, due to the costliness and complexity of these methods, the highly efficient nonclassical theories are used instead.

Due to the inefficiency of the classical continuum theory in accurately predicting the mechanical properties and dynamic behavior of micro/nanostructures with minute dimensions, continuum theories accounting for the length scale parameter and the nonlocal parameter are used to study these structures. In recent years, numerous studies have attempted to investigate the size effects using various theories in tackling various problems. To account for size effects, those studies have used the material length scale parameter in higher order continuum theories [2–28] and the nonlocal constant in the nonlocal theory [29–50].

The size-dependency of the mechanical properties of micro/nanostructures as well as the substantial effect of the size parameter on these properties in the nanoscale has given considerable appeal to the higher order continuum theories in recent years, since these theories incorporate the material length scale parameter into the study of micro/nanostructures. In higher order continuum theories, strain energy is dependent on strain as well as strain variation [51–54]. Mindlin rewrote the higher order stress theory which incorporates the higher order strain gradient [55]. He rewrote the equations for center-symmetric isotropic objects, in which case strain energy has five linear elastic constants incorporating the material length scale parameters. Aifantis simplified Mindlin's strain gradient theory by using one material length scale parameter rather than five [56].

In the nonlocal theory, stress at the reference point is dependent not only on strain at the reference point but also on strain at other points in the object. This theory predicts rigidity to be lower. Many researchers have used the nonlocal theory to investigate the behavior of nanostructures. For instance, Ansari et al. computed the nonlocal constant for double-walled carbon nanotubes (DWCNTs) using the MD method. They investigated the vibration of DWCNTs using the shell theory [57]. Hu et al. computed the vibration of SWCNTs using the nonlocal theory and determined the size effect of SWCNTs by comparing their results to the MD results [58]. Ansari and Sahmani examined the vibration of SWCNTs using the MD method and the beam model and determined the

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nonlocal constant for those SWCNTs using the nonlocal theory [59]. Hu et al. investigated wave propagation in SWCNTs and DWCNTs. They accounted for the nonlocal constant of the carbon nanotube using the nonlocal theory and computed the value of the nonlocal constant for the two types of carbon nanotubes by comparing their results to the MD results [60].

Zenkour and Sobhy investigated the thermal buckling of a nanoplate using the nonlocal theory. They placed the nanoplate on a Pasternak foundation and investigated the effect of the nonlocal constant, foundation stiffness, and aspect ratio on nanoplate buckling [61]. Sobhy and Radwan investigated the vibration and buckling of FGM nanoplates by considering the thermal effects. They used the new quasi-3D hyperbolic plate theory to model the plates and employed the nonlocal theory to account the size effects. They demonstrated that an increase in mode number is accompanied by a increase in natural frequency, and a decrease in power law index, aspect ratio and nonlocal parameter is accompanied by an increase in natural frequency [62].

Today, considering the greater precision of calculation in 3-D space, many researchers are interested in investigating the dynamic behavior of micro- and nano-structures using the shell theory. By way of example, Ansari et al. investigated nanotube vibration using the Donnell shell theory and accounted for the size effect using the nonlocal theory. Furthermore, they compared the results of the shell model with those of the beam model, demonstrating that the natural frequencies in the latter are greater than those in the former, which is due to the presence of the circumferential mode [63]. Ghorbanpourarani et al. investigated the transverse vibration of a single-walled nanotube and a double-walled nanotube using the beam and cylindrical shell models. They used the Timoshenko and Euler-Bernoulli models as beam models and the Donnell shell theory as the shell model, demonstrating that the frequencies in the cylindrical shell model are smaller than those in the two beam models, which is due to the three-dimensional quality of the cylindrical shell model [64].

As demonstrated by the above-mentioned facts, the nonlocal and higher order continuum theories offer opposite predictions of the physical and mechanical behaviors of micro/nanostructures, and many studies have examined the simultaneous impacts of nonlocal constant and material length scale parameter on wave propagation in viscoelastic nanostructures. During deformation, viscoelastic materials exhibit both elastic and viscose behaviors. They are used for noise damping and shock absorption. The elastic modules are modeled as spring and the viscose modules are modeled as dampers. Tang et al. investigated wave propagation in viscoelastic SWCNTs. They used the Timoshenko beam to model the SWCNTs and the nonlocal strain gradient theory to account for size effects. They examined the impacts of wavenumber, SWCNT radius, nonlocal constant, material length scale parameter, and damping constant on phase velocity and damping ratio [65]. Ebrahimi and Barati investigated wave propagation in a heterogeneous viscoelastic nanobeam using the nonlocal strain gradient theory. They used the Euler-Bernoulli beam to model the SWCNT and demonstrated that variation of wavenumber, nonlocal constant, and material length scale exerts a considerable impact on SWCNT phase velocity [66]. Tang et al. studied the effect of material length scale parameter and nonlocal constant on wave propagation in viscoelastic SWCNTs using the nonlocal strain gradient theory, demonstrating that the damping constant, Winkler constant, and SWCNT diameter exert a considerable impact on SWCNT phase velocity [67]. Ebrahimi and Barati investigated wave propagation in an FGM thermoelastic nanoplate using the nonlocal strain gradient theory, showing that variation of temperature, wavenumber, nonlocal constant, and material length scale parameter exerts a remarkable impact on the phase velocity of the FGM nanoplate [68]. Using the nonlocal strain gradient theory, Li et al. examined wave propagation in a viscoelastic SWCNT subjected to a magnetic field and incorporated the surface effects into the model. They investigated the effect of the nonlocal constant, material length scale parameter, magnetic field, and damping constant on phase velocity and damping ratio

[69]. Using the nonlocal strain gradient theory, Li and Hu investigated wave propagation in fluid-conveying viscoelastic SWCNTs, demonstrating that the effects of the nonlocal constant, material length scale parameter, fluid velocity, and damping ratio on phase velocity and damping ratio are considerable [70].

According to the foregoing discussion, sufficient attempts have yet to be made to investigate the effects of the length scale and nonlocal constant on viscoelastic SWCNTs using the cylindrical shell model. For this reason, in the present study, wave propagation in cylindrical viscoelastic SWCNTs is investigated using the nonlocal strain gradient theory. To this end, the thin shell theory is employed. The classical governing equations are derived using Hamilton's principle. The Kelvin–Voigt model is used to model the viscoelasticity, and the SWCNT is surrounded by a medium of viscoelastic foundation. To validate the results, wave propagation of a SWCNT is compared with that obtained through the MD method by Ref. [60]. In the Results section, the impacts of the nonlocal constant, material length scale parameter, damping constant, circumferential wavenumber, and radius on phase velocity in viscoelastic SWCNTs are investigated.

2. Governing equation of problem

2.1. Classical formulation

The strain energy for a linear elastic material in infinitesimal deformation is expressed as:

$$U_s = \frac{1}{2} \int_{\Omega} \sigma_{ij} \varepsilon_{ij} dV \tag{1}$$

where

$$\varepsilon_{ij} = \frac{1}{2} (\partial_i U_j + \partial_j U_i) \tag{2}$$

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} \tag{3}$$

In the above equations, σ_{ij}, ε_{ij}, U_i and C_{ijkl} represent the Cauchy stress tensor, small strain tensor, displacement vector, and fourth order elasticity tensor, respectively. Considering plane stress assumption in the cylindrical shell theory and isotropic materials, stress-strain relations are expressed as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{x\theta} \end{Bmatrix} = \begin{pmatrix} Q_{11}\left(1 + g\frac{\partial}{\partial t}\right) & Q_{12}\left(1 + g\frac{\partial}{\partial t}\right) & 0 \\ Q_{12}\left(1 + g\frac{\partial}{\partial t}\right) & Q_{22}\left(1 + g\frac{\partial}{\partial t}\right) & 0 \\ 0 & 0 & Q_{66}\left(1 + g\frac{\partial}{\partial t}\right) \end{pmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ 2\varepsilon_{x\theta} \end{Bmatrix} \tag{4}$$

where

$$Q_{11} = Q_{22} = \frac{E}{1 - \nu^2}, \ Q_{12} = \frac{E\nu}{1 - \nu^2}, \ Q_{66} = \frac{E}{2(1 + \nu)} \tag{5}$$

In the Eqs. (4) and (5), g, G, E and ν are the viscous damping coefficient, shear modulus, Young's modulus, and Poisson's ratio, respectively.

According to the thin shell theory, the displacement components of the viscoelastic SWCNT can be expressed as

$$U(x, \theta, z, t) = u(x, \theta, t) - z\frac{\partial w(x, \theta, t)}{\partial x},$$

$$V(x, \theta, z, t) = \left(1 + \frac{z}{R}\right)v(x, \theta, t) - \frac{z}{R}\left(\frac{\partial w(x, \theta, t)}{\partial \theta}\right)$$

$$W(x, \theta, z, t) = w(x, \theta, t) \tag{6}$$

where u, v and w represent displacement vectors in the middle plane of viscoelastic SWCNT which are along the x, θ and z axes, respectively. Also t represents time (Fig. 1).

By substituting Eq. (6) in Eq. (2), the components of classical strain are obtained as follows