Educational Research Review 24 (2018) 84–97

Contents lists available at ScienceDirect

Educational Research Review

journal homepage: www.elsevier.com/locate/edurev

Methodological Reviews

The use of Latin-square designs in educational and psychological research

John T.E. Richardson

Institute of Educational Technology, The Open University, Walton Hall, Milton Keynes MK7 6AA, United Kingdom

ARTICLE INFO

Keywords:
Educational research
Experimental design
Latin squares
Psychological research

ABSTRACT

A Latin square is a matrix containing the same number of rows and columns. The cell entries are a sequence of symbols inserted in such a way that each symbol occurs only once in each row and only once in each column. Fisher (1925) proposed that Latin squares could be useful in experimental designs for controlling the effects of extraneous variables. He argued that a Latin square should be chosen at random from the set of possible Latin squares that would fit a research design and that the Latin-square design should be carried through into the data analysis. Psychological researchers have advanced our appreciation of Latin-square designs, but they have made only moderate use of them and have not heeded Fisher's prescriptions. Educational researchers have used them even less and are vulnerable to similar criticisms. Nevertheless, the judicious use of Latin-square designs is a powerful tool for experimental researchers.

1. Introduction

A Latin square is a particular kind of configuration of integers, letters of the alphabet, or other symbols. Latin squares have been of interest to mathematicians for a very long time. However, Fisher (1925) proposed that they could also be very useful in experimental research for controlling the effects of extraneous variables. To be used properly, he argued that a Latin square needed to be chosen strictly at random from the universe of possible Latin squares that would fit a particular research design. He also insisted that the Latin-square design should be carried through into the analysis of the results.

Fisher was interested in agricultural experiments, but researchers in other fields came to realise that Latin-square designs could be useful in their work. This is notably the case in medical research, where it is nowadays widely recognised that Latin-square designs provide an efficient and effective way of controlling for the effects of extraneous variables, especially the effects of temporal order or sequence in repeated-measures designs. On February 7, 2017, the bibliographic database MEDLINE recorded a total of 4055 publications since 1948 that contained the phrase “Latin square” in their titles, abstracts, keywords, or metadata, yielding an average of 58.8 such publications per year over the relevant 69-year period.

Educational researchers also realised that Latin squares could be used in their work, but Latin-square designs do not appear to have been adopted in education anywhere near as often as in medical research. Informal enquiries suggest that nowadays many educational researchers are unaware of their existence, and that their students do not learn about these designs in the course of their training. This is exceedingly unfortunate, because educational researchers may be missing the opportunity to exploit a potentially valuable tool in the design of their experiments. Accordingly, my aim in this article is to advocate the more widespread use of Latin-square designs in educational research.

To achieve this, I first review the history of Latin squares and their potential role in experimental research. I note that Latin-square designs are more efficient and hence more powerful than reasonable alternatives such as completely randomised designs or

E-mail address: John.T.E.Richardson@open.ac.uk.

https://doi.org/10.1016/j.edurev.2018.03.003
Received 12 June 2017; Received in revised form 12 March 2018; Accepted 20 March 2018
Available online 24 March 2018
1747-938X/

1	2	3	4
2	3	4	1
3	4	1	2
4	1	2	3

Fig. 1. A 4 × 4 Latin square.

randomised complete block designs. I then review how educational researchers have made use of Latin-square designs in their experiments. In fact, such designs have been more widely adopted in psychological experiments, and so I also review how psychological researchers have made use of Latin-square designs. I compare research practice in these two disciplines with a focus on whether they have complied with Fisher's stipulations regarding the use of Latin-square designs in experiments. I conclude by advocating the more formal and rigorous use of Latin-square designs in future educational research.

2. What is a Latin square?

A Latin square is a grid or matrix containing the same number of rows and columns (k, say). The cell entries consist of a sequence of k symbols (for instance, the integers from 1 to k, or the first k letters of the alphabet) inserted in such a way that each symbol occurs only once in each row and only once in each column of the grid. Probably the best known modern examples are Sudoku puzzles, which will be discussed in Section 2.1. As a simpler example, Fig. 1 shows a Latin square with four rows and four columns that contains the integers from 1 to 4. Fig. 1 is an example of a standard form (also known as a reduced or normalised Latin square), in that the numbers in the first row and the numbers in the first column are in their natural order. Nonstandard Latin squares can be derived from standard forms by interchanging different rows in the grid, by interchanging different columns in the grid, or by doing both of these.

Latin squares are sometimes discussed in connection with magic squares. A (normal) magic square with k rows and k columns contains just one occurrence of each of the integers from 1 to k^2 in such a way that the numbers in each row, each column, and each diagonal add up to the same total (which simple mathematics shows must be $k(k^2 + 1)/2$). For example, a magic square with four rows and four columns would contain just one occurrence of each of the integers from 1 to 16, and the numbers in each row, column and main diagonal would all add up to $[4 \times (16 + 1)]/2$ or 34. (Non-normal magic squares can be constructed using more complex arithmetic progressions than 1, 2, …, k^2.) Latin squares and normal magic squares are conceptually different structures, but they are related in that Latin squares can be used to construct magic squares of the same dimensions (Emanouilidis, 2005).

2.1. A brief history of Latin squares

Kendall (1948) speculated that games and puzzles based upon Latin squares might have been first devised following the introduction of playing cards into Western Europe, which occurred during the 14th Century. In fact, Latin squares had been described by Arab and Hindu mathematicians prior to this and are depicted in early spiritual motifs found in the Middle East and India as well as Catalonia (Andersen, 2013).

Even so, the earliest written account in the West seems to be contained in a collection of puzzles and "recreations" by a French mathematician, Jacques Ozanam. The first edition had been published in two volumes in 1694; it described magic squares but not Latin squares. Ozanam died in 1718, but a "new edition, revised, corrected and augmented" was published posthumously as four volumes in 1723[1]. In a section entitled "Various Amusing Tricks", Ozanam (1723, p. 434) showed how to arrange the four kings, the four queens, the four jacks, and the four aces in a pack of cards in a 4 × 4 array so that there was a king, a queen, a jack, and an ace in each of the four rows, in each of the four columns, and in both of the main diagonals, and so that there was a spade, a club, a heart, and a diamond in each of the four rows, in each of the four columns, and in both of the main diagonals. Fig. 2 shows the solution presented by Ozanam (1723, Plate 12, Fig. 35), but there are other solutions.

Ozanam's example goes beyond the simple notion of a Latin square in two respects. First, the solution involves not one but two Latin squares: one specifies the arrangement of the ranks (kings, queens, jacks and aces); the other specifies the arrangement of the suits (spades, clubs, hearts and diamonds). The two squares are orthogonal to each other, in the sense that each possible combination of ranks and suits only occurs once. Later, Euler (1782) discussed similar examples, using Latin (i.e., Roman) letters for the symbols in the first Latin square and Greek letters for the symbols in the second Latin square, and these became known as "Graeco-Latin" squares. Nowadays, however, they are more commonly referred to just as "pairs of orthogonal Latin squares" (or, occasionally, as "Eulerian squares").

Second, in Ozanam's solution, each symbol occurs only once in each of the main diagonals as well as in each of the rows and each of the columns. If the four ranks in the solution are represented as jacks = 1, aces = 2, kings = 3, and queens = 4, then the first of the

[1] Some authors cite this work as "Ozanam (1723)". At the time, it was common to show the date of printing on the title page of a book rather than its date of publication. The authors in question appear to have obtained reprints of this work from 1725.