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Learning angles through movement: Critical actions for developing understanding in an embodied activity

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ABSTRACT

Angle instruction often begins with familiar, real-world examples of angles, but the transition to more abstract ideas can be challenging. In this study, we examine 20 third and fourth grade students completing a body-based angle task in a motion-controlled learning environment using the Kinect for Windows. We present overall pre- and post-test results, showing that the task enhanced learners' developing ideas about angles, and we describe two case studies of individual students, looking in detail at the role the body plays in the learning process. We found that the development of a strong connection between the body and the abstract representation of angle was instrumental to learning, as was exploring the space and making connections to personal experiences. The implications of these findings for developing body-based tasks are discussed.

1. Introduction

Like many concepts in geometry, developing a thorough understanding of angle is challenging for many students (Clements & Battista, 1992). Students have difficulties with angle measurement (Freudenthal, 1983), operations with angles (Usiskin, 1987), and recognizing angles in different contexts (Mitchelmore, 1998). Yet, angles are the foundation for much of geometry and important for students to understand at an early age (Menon, 2009). Further, the lack of a solid understanding of angle concepts can make understanding and working with trigonometric functions more difficult later on (Moore, 2013).

A rich history of research on the instruction of angle concepts documents a wide variety of tasks used to support learning. These range from Piaget and Inhelder's (1956) task in which children examine lazy tongs to students creating their own protractors (Krainer, 1993) to Logo programming activities (Clements & Battista, 1989, 1990). More recently, building off of the work with Logo, the use of the body and physical situations to teach angles has been explored (Devichi & Munier, 2013; Fyhn, 2008; Touval & Westreich, 2003). There is increasing evidence that body-based activities can be effective at developing mathematical understanding (Howison, Trninic, Reinholz, & Abrahamson, 2011; Petrick & Martin, 2012; Wright, 2001). However, Mitchelmore and White (2000) hypothesize that while children begin to build their understanding of angles as early as preschool based on their everyday life experiences, building an abstract concept of angle is difficult and takes time and support.

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To support this transition, we developed a task that uses a Kinect for Windows program to detect angles students make with their bodies while simultaneously projecting an abstract, visual representation of the angle formed by two arrows meeting at a point on the screen. We hypothesize that this design will support the development of a connection between the physical and abstract representations and that students' actions and their ideas about angles will develop together. The purpose of the present study was to examine the impact of the task on learning and to explore characteristics of students' actions during the task that supported or interfered with developing an understanding of angle and angle measurement.

2. Background

2.1. Angle definitions and types of representations

Part of the reason angles are difficult to understand may stem from the fact that a variety of definitions are used. Henderson and Taimina (2005) describe three common ways of defining angle: an angle as a geometric figure (a pair of rays with a common endpoint), an angle as a dynamic figure (a turn or rotation), and an angle as a measure. An understanding of angle as a complex concept incorporating all three definitions is slow to develop over time (Casas-García and Luengo-González, 2013; Lehrer, Jenkins, & Osana, 1998).

An angle defined as a geometric figure can be depicted using a static angle representation, while defining an angle as a turn or rotation requires a dynamic representation. Both static and dynamic representations have different benefits and limitations for learners (Casas-García and Luengo-González, 2013). In particular, static representations make it difficult to depict certain angles (e.g. an angle of 0°) and to visualize angles in a wide variety of positions. Dynamic representations often lack the visual or physical support learners need (Mitchelmore & White, 2000). For example, when conceptualizing the angle created by the turning of a tire on an axle, it is challenging to keep track of where the angle begins and ends as there is nothing to indicate the starting position of the tire before the turn. Devichi and Munier (2013) conducted a study comparing instructional sequences using static and dynamic angle representations in third and fourth grade classrooms. They found that students who followed the dynamic instructional sequence drew more non-prototypical angles on a post-test than the students who learned with static angles, but they found no differences between groups on an angle variation task in which they were asked to draw angles of different sizes. Wilson and Adams (1992) suggest learners need to integrate ideas from both static and dynamic angle representations as their understanding of angle grows.

2.2. Development of angle understanding

Typical angle instruction at the elementary level focuses on static representations of angles depicted on the board or on worksheets. Students learn to identify the parts of an angle, measure angles, and classify them. Given the multifaceted nature of angles, working with angles in such an abstract way early on can be challenging for students. Mitchelmore and White (2000) propose that angle concepts develop through a process of progressive abstraction and generalization beginning with children's experiences with concrete examples of angles. They identified 14 different types of physical angle situations children may encounter in their daily life such as angles formed by scissors or the hands of a clock (Mitchelmore & White, 1998). As students begin to see similarities between angles in different situations, they begin to develop an abstract angle concept (Mitchelmore & White, 2000). The most common and abstract conception of angle that children construct is that of two rays with a common endpoint.

Several misconceptions about angles persist among students and must be addressed in instruction. Students frequently fail to recognize that two angles are the same measure if they are oriented in non-standard directions (Browning, Garza-Kling, & Sundling, 2007; Noss, 1987). For example, Outhred (1987) found that children considered the orientation of an angle when determining its size. Students also tend to attribute the size of an angle to the length of the line segments that make up the angle (Clements, 2003; Clements & Battista, 1989; Fyhn, 2008). This misconception can stay with students over an extended period of time. For example, Lehrer et al. (1998) conducted a longitudinal study looking at children's conceptions of angle, and they found that students made decisions about the size of an angle based on the length of the line segments forming angles and that this misconception persisted through the three year study.

2.3. Instructional tasks

A variety of instructional tasks have been implemented in research examining angles, and the results of these studies offer insight into important design features. Douek (1998) suggests that learning about angles through body-based activities plays an important role in the development of their understanding of angles. When students use their bodies to act out angle concepts, it provides an opportunity for students to draw connections between their physical movements and mathematics concepts. The research with Logo provides one example of students creating angles with their bodies. Clements, Battista, Sarama, and Swaminathan (1996) used Logo and activities in which students acted out the moves of the Logo turtle in order to develop a relationship between "turn-as-body-motion" and "turn-as-number." While acting out the turns was helpful in some respects, students experienced difficulty with conceptualizing angles in terms of rotations because the turning movement did not leave a trace. Consequently, students would have to coordinate the start and endpoints of the turn