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ScienceDirect

Cognitive Systems Research 52 (2018) 275–290

Cognitive Systems RESEARCH

www.elsevier.com/locate/cogsys

Equations of mind: Data science for inferring nonlinear dynamics of socio-cognitive systems

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Received 14 April 2018; received in revised form 15 June 2018; accepted 30 June 2018

Abstract

Discovering the governing equations for a measured system is the gold standard for modeling, predicting, and understanding complex dynamic systems. Very complex systems, such as human minds, pose stark challenges to this mode of explanation, especially in ecological tasks. Finding such "equations of mind" is sometimes difficult, if impossible. We introduce recent directions in data science to infer differential equations directly from data. To illustrate this approach, the simple but elegant example of sparse identification of nonlinear dynamics (SINDy; Brunton, Proctor, & Kutz, 2016) is used. We showcase this method on known systems: the logistic map, the Lorenz system, and a bistable attractor model of human choice behavior. We describe some of SINDy's limitations, and offer future directions for this data science approach to cognitive dynamics, including how such methods may be used to explore social dynamics.

1. Introduction

Differential equations define the time evolution of a dynamical system. Their precision inspires some to see such mathematical formulation as critical to scientific understanding. This perspective on differential equations found prominent expression in the dynamical systems approach to cognition of the 1990s (Port & Van Gelder, 1995; Van Gelder, 1995), and was the subject of vigorous debate (Bechtel, 1998; Eliasmith, 1996): "Dynamical systems governed by differential equations are a particularly interesting and important subcategory, not least because of their central role in the history of science." (Van Gelder, 1995, p. 368) Simon (1992) famously expressed an even stronger position, arguing that cognitive explanation is founded on "difference equations" which characterize much cognitive systems research still:

> "For systems that change through time, explanation takes the form of laws acting on the current state of the system to produce a new state – endlessly. Such explanations can be formalized with differential or difference equations. A properly programmed computer can be used to explain the behavior of the dynamic system that it simulates. Theories can be stated as computer programs." (Simon, 1992, p. 160)

Nowadays this mode of mathematical description and explanation permanently inhabits many realms of cognitive science.[1] It was well established even before this recent

[1] A reviewer helpfully pointed out that description and explanation should not be confounded, and that equations alone rarely fulfill our conventional notions of "explaining" systems. For simplicity, we do not distinguish between these modes of scientific inquiry – describing and explaining – but assume that differential equations are considered, by a great many, to be important for both.

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https://doi.org/10.1016/j.cogsys.2018.06.020

debate. From the firing of single nerve cells (Hodgkin & Huxley, 1952) and the control of an entire physical body (Beek, Turvey, & Schmidt, 1992; Kugler, Kelso, & Turvey, 1980) to multi-agent models (Richardson et al., 2016), systems of differential equations have long captured a wide variety of psychological phenomena. When we have a set of differential equations for a system, we can predict its time evolution, understand its controlling variables, and identify how system variables interact. These dynamic equations can also participate with other forms of cognitive explanation, such as *mechanistic* explanations of how a cognitive architecture is composed of various particular parts and their interactions (Kaplan & Bechtel, 2011).

Despite their power, differential equations are not always easy to identify. Identification of governing equations can involve an interacting cycle of mathematical invention and empirical tinkering. Guided by intuition, a scientist can happen upon a formulation that generates a covering law (Hempel, 1966). Consequences of this covering law can be explored to consider other formulae in other domains of application. The literature on this is deep and colorful, and excellent reviews of the philosophy and history of science abound (Brush, 1974; Hempel, 1966; Hirsch, 1984; Kuhn, 1962).

Cognitive scientists continue to study and model this psychological process of identifying scientific generalizations and natural law (Addis, Sozou, Lane, & Gobet, 2016; Klahr & Simon, 1999; Langley, 1987). A complementary approach, made possible by computational tools of the day, is to use data and algorithms together to automatically recover dynamical laws. This is what we consider here in this paper. There is an emerging domain, growing rapidly with the advent of data science and machine learning, to precisely recover differential equations from raw data. This offers considerable potential to researchers interested in the dynamics of socio-cognitive systems. It may be possible to use these tools for new and explicit descriptions of system dynamics, even when the data are noisy, and especially when there are plenty of data to be found (a common circumstance these days: Paxton & Griffiths, 2017).

There has been considerable prior work on equation discovery. Motivated by the same points we raise above, researchers over the past two decades have explored different frameworks for automatic recovery of governing equations. Below we first briefly review this past work through influential examples. After this, we introduce a recent simple and elegant formulation of equation discovery (SINDy; Brunton, Proctor, & Kutz, 2016). Based only on transformation of time series data, and simple sparse regression, a researcher can recover equations for their measured systems. In some simple cases, these equations may reflect a full reconstruction of a system's underlying dynamics. More complex cases present other challenges, but in these more complex situations SINDy may still be useful. Below, we introduce SINDy and then showcase how it works on a number of example systems. We also outline its key limitations. After this, we summarize a few outstanding issues in these domains, including how SINDy and related methods could be expanded in the future to help recover governing equations of social systems.

1.1. Some background

There has been considerable prior work on equation discovery. Classic work in cognitive science itself can be found in Langley (1981), who used symbolic cognitive models to infer equations from data. His early model, BACON.3, is meant to capture some important aspects of human scientific activity. More recently, Langley and colleagues (Langley, Sanchez, Todorovski, & Dzeroski, 2002) have also used time series data in an Inductive Process Modeler that can fix certain parameters on population dynamics models. These general approaches fall under the rubric of symbolic machine learning, as a kind of heuristic search. For example, process models of biological systems can include a space of parameters that describe the relationship among variables (Džeroski & Todorovski, 2008). A heuristic search navigates this parameter space under certain constraints to best fit a set of data.

Crutchfield, Shalizi, and others have developed a hidden Markov approach that generates a directed graph that represents a theory of a system from a time series of its behavior (Crutchfield, 1994, 2011; Shalizi & Crutchfield, 2001; Shalizi & Shalizi, 2004). This framework finds transitions between system states in coarse-grained representation of the time series. The result is a kind of compact theory which can describe the time evolution of the system. It also provides descriptive measures of the system, such as its computational complexity. This modeling framework can be used to simulate the relationship between measurement level and theory, and can be likened to a cognitive agent seeking to explain and model a system's dynamics (Crutchfield, 1994; Dale & Vinson, 2013).

There are many related techniques, both in cognitive science and in other realms of the physical sciences. An excellent review can be found in Sozou, Lane, Addis, and Gobet (2017). Much work used clever analysis of time series with *assumed* form of laws to recover particular systems (Bezruchko, Karavaev, Ponomarenko, & Prokhorov, 2001; Bünner, Meyer, Kittel, & Parisi, 1997; Crutchfield & McNamara, 1987; Smith, 1992).

With the advent of large matrix libraries, advanced regression methods are now possible. Schmidt and Lipson (2009) use symbolic regression and motion tracking of physical systems to derive various equations of motion. Example systems included chaotic systems, such as double pendula. Their approach involves extraction of motion time series, and then seeking invariances (correlation structure) among the measured variables according to candidate symbolic functions. The symbolic functions are found via a search through a space of candidates, generated randomly and gradually winnowed down based on best fit (see their Fig. 2). This method is closely related to the one we showcase below, with the primary difference that in SINDy