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Distributed optimization for multi-agent systems with constraints set and communication time-delay over a directed graph

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ABSTRACT

This paper is concerned with the problem of distributed optimization for a multi-agent system with constraints set and communication time-delay over a directed graph. The considered cost function is a summation of all local cost functions associated with each agent. Firstly, a novel distributed algorithm is developed to solve such a problem, where auxiliary state variables are also exchanged to compensate the nonzero gradient of local cost function and accelerate the convergence of estimate states to the optimal point. Secondly, the minimizer of distributed optimization of a multi-agent network is determined by the variational inequality in spite of the existence of time delay. Furthermore, delay-dependent and delay-free sufficient conditions on the convergence of states of agents to the optimal point are derived by constructing a new Lyapunov–Krasovskii functional, respectively. Finally, a numerical example and a comparison are provided to validate the obtained results.

1. Introduction

A distributed strategy is an important approach in the research field of a multi-agent system, where distributed optimization has attracted more and more attentions in science and engineering owing to its extensive applications. A distributed optimization algorithm has many advantages such as reliability, scalability and the ability of reducing communications. This is also why many researchers explore distributed optimization algorithms. Examples include distributed cooperative control [12,26,39], distributed sensor fusion [3,15], distributed parameter estimation [10,22], distributed impulsive control [19] and distributed resource allocation over systems ([5,47,48], and the references therein), to list a few. Ge et al. [13] summarizes the composition of the networked system, the technical constraints nowadays and the commonly used methodologies to deal with such networked problems, and gives future research directions and suggestions for multi-agent systems.

The goal of distributed optimization is to seek for minimization or maximization of a cost function via cooperation of the agent with its neighbors, where the cost function is a summation of local cost functions of all agents and each agent has only accesses to its own local cost function and a local decision variable. In recent years, distributed optimization problems are extensively studied based on multi-agent systems ([24,28,32,46,49] and the references therein). The purpose of introducing a multi-agent system is to deal with distributed optimization utilizing the rich theory of a multi-agent system. A large

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number of algorithms for distributed optimization based on a multi-agent system have been established and analyzed. Generally, they are divided into two categories [46]: 1) discrete-time approaches and 2) continuous-time approaches. In most of the recent work, distributed optimization methods are developed in the discrete-time framework [1,28,32,33,35,50]. The incremental subgradient algorithm as an earlier method addresses distributed optimization problems with constraints (unconstraints) in [35]. The optimization problem without constraints set on multi-agent networks is investigated in [32], where the subgradient-based method is proposed to solve an optimization problem over a time-varying network. This approach is further extended to the projected subgradient algorithm in [33] for an identical constraints set and the case of different constraints set is explored in [28]. A consensus-based distributed primal-dual subgradient method is developed to deal with optimization problem with a global inequality constraints in [50]. Whereafter, the extended or modified algorithms are also applied to distributed optimization with equality constraints and inequality constraints in [29,44].

In some practical systems, states of each agent are continuous-time dynamic, which demands design of a continuous-time algorithm to solve distributed optimization for a multi-agent system and convergence of the proposed algorithm can be analyzed through the classical Lyapunov method. Therefore, a lot of work have imported continuous-time methods to handle the most popular distributed optimization based on multi-agent systems. In [14], a consensus-based continuous-time algorithm is proposed for distributed optimization over a directed topology. This algorithm is further expanded in [23] and two different communication mechanisms are presented to remit communication pressure among agents. In [38], the gradient-based method is designed to solve such a problem with nonlinear dynamics. A continuous-time gradient-based approach is established to deal with an optimization problem with a common constraints set in [30].

Distributed optimization based on multi-agent systems is implemented via the local communication, local calculation and cooperation among agents. When the size of the communication network among agents is relatively large, it is necessary to consider the impact of communication time-delay over networks on the optimization performance since communication time-delay is inevitable [36]. For example, in an electronic circuits, it has been shown that time-delay is an important source of crystal oscillator [25]. Consensus (average consensus) is an important research topic for multi-agent systems [6,7,7,11,17,18,20,23,30] and a lot of work are reported on such a topic with communication time-delay [21,27,42,51], where the influences of time-delay on consensus are studied. In addition, large time-delay causes the system of interest to crash or disintegrate [45]. So far, the communication time-delay is widely investigated in discrete-time systems [40,41,43], and the references therein. Few references are reported on considering time-delay in distributed optimization of continuous-time systems except [46] where distributed optimization of multi-agent systems with communication time-delay is studied and slow-delay and fast-delay are involved as well.

Inspired by Yang et al. [46], consensus and optimality analysis are further explored in continuous-time multi-agent systems for distributed optimization. The objective of this paper is to solve distributed optimization of multi-agent systems with constraints set and communication time-delay. In order to seek for the network optimizer, a control protocol is developed to find the optimal estimate and exchange the estimate with its neighbors with the consideration of communication time-delay. Therefore, it is significant to explore the role of time-delay on distributed optimization of multi-agent networks. The main contributions of this paper are emphasized as follows:

- A novel distributed algorithm is developed to solve the problem of distributed optimization with constraints set and communication time-delay over a directed graph, where the auxiliary states for compensating the nonzero gradient of local cost function are also exchanged and thus accelerate the convergence to the minimizer of a multi-agent network.
- Delay-dependent sufficient conditions on guaranteeing the convergence of states of agents to the optimal point are derived using a new Lyapunov–Krasovskii functional and the variational inequality. Delay-free sufficient conditions are also obtained by a similar approach.

The rest of this paper is organized as follows. In Section 2, preliminaries and problem formulation are presented. In Section 3, a distributed consensus-based algorithm with communication time-delay and optimality analysis are established. Convergence analysis of network dynamics is given in Section 4. A numerical example and a comparison are provided to testify the theoretical results in Section 5. Finally, Section 6 summarizes the investigation and gives the direction of future research.

Notation. B and B^T denote a matrix and its transpose with appropriate dimensions. $[D]_{ij}$ denotes the ith row and the jth column entry of the matrix D. R denotes the set of real numbers, R^n denotes n dimensional real vector space. 1_n, 0_n denote n dimensional vector with all 1 and 0 elements, respectively. Denote $col(x_1, x_2, \dots, x_n) = \left[x_1^T, x_2^T, \dots, x_n^T\right]^T$ as the column vector stacked with vectors $x_1, x_2, \dots, x_n$. $diag(x_1, x_2, \dots, x_n)$ is the diagonal matrix with the ith diagonal entry $x_i, i = 1, 2, \dots, n$. I_n is the identity matrix with n dimensions. The gradient vector of a function f is denoted as ∇f. $P_\Omega(\cdot)$ as projection on domain Ω. $J \otimes H$ denotes the Kronecker product of matrix J and H. The symmetric element of a symmetric matrix is denoted by $*$. $W < 0$ means that W is a negative definite and symmetric matrix. Generally, $x(t)$ is recorded x_t.

2. Preliminaries and problem formulation

2.1. Graph theory

The following concepts of graph theory can be found in [31]. A graph is defined as $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, 3, \dots, n\}$ is a set of agents and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges. Agent j is a neighbor of agent i if agent i can receive information from agent