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Exponential Wavelet Iterative Shrinkage Thresholding Algorithm for compressed sensing magnetic resonance imaging

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ABSTRACT

It is beneficial for both hospitals and patients to accelerate MRI scanning. Recently, a new fast MRI technique based on CS was proposed. However, the reconstruction quality and computation time of CS-MRI did not meet the standard of clinical use. Therefore, we proposed a novel algorithm based on three successful components: the sparsity of EWT, the rapidness of FISTA, and the excellent tuning in SISTA. The proposed method was dubbed Exponential Wavelet Iterative Shrinkage/Threshold Algorithm (EWISTA). Experiments over four kinds of MR images (brain, ankle, knee, and ADHD) indicated that the proposed EWISTA showed better reconstruction performance than the state-of-the-art algorithms such as FCSA, ISTA, FISTA, SISTA, and EWT–ISTA. Moreover, EWISTA was faster than ISTA and EWT–ISTA, but slightly slower than FCSA, FISTA and SISTA.

1. Introduction

In conventional data acquisition and signal reconstruction, the number of data points sampled per unit of time is governed by the Nyquist–Shannon Sampling Theorem. This theorem states that a band-limited analog signal can be perfectly reconstructed from a finite sequence of samples if the sampling rate exceeds $2B$, where B is the highest frequency of the original signal [23]. When the rate of sampling is lower than the Nyquist frequency, the high frequency portion of signals will be aliased or folded over into the lower frequency portion of the signal [50]. In magnetic resonance imaging (MRI), the signals are not band-limited and the acquisition interval of k-space data points [12], Δk, is determined by 1/FOV, where FOV represents the desired field of view [8]. When the sampling rate is lower than 1/FOV, the reconstructed image will contain fold-over artifacts and portions of the object that are outside of the FOV will appear in the image. However, acquiring a full set of data points in k-space at a rate that avoids image fold-over can be time-consuming [49].

Recently, a new fast MRI technique based on compressed sensing (CS) was developed that permits the acquisition of a small number of k-space data points at random rates lower than 1/FOV, and the reconstruction of the image using nonlinear

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constrained minimization algorithms [27]. The random undersampling employed in compressed sensing magnetic resonance imaging (CS-MRI) produced massive and random aliasing, and thus the reconstruction of the CS-MRI can be a possible procedure for anti-aliasing the image [51].

Sparse representation is one of the most important components of CS-MRI. Wavelet transform (WT) is powerful in representing images in a sparse way. Curvelet was reported to perform better in representing curve-like features of images [28]. Contourlet is similar to curvelet, and it is capable of capturing directional information [15]. However, curvelet and contourlet transforms hinder computation resources. In past research, we found that the exponent of wavelet transform (EWT) can increase the sparsity of Magnetic Resonance (MR) images significantly and reduce the running time [53].

On the other hand, reconstruction algorithm is another important component of CS-MRI. Total Variation (TV) employs the sum of Euclidean norms of the gradient, choosing finite differences as the sparsifying transform. TV is thus only suitable for piecewise-constant objects. The Iterative Shrinkage/Thresholding algorithm (ISTA) is a nonparametric method that is proven to converge mathematically. However, the slow convergence prevents its widespread use [7]. Fast ISTA (FISTA) exploits the result of past iterations to speed up convergence [3]. Subband adaptive ISTA (SISTA) optimizes wavelet-subband-dependent parameters [2].

In this study, we worked on the possibility of combining and tailoring the EWT, FISTA, and SISTA. We termed the new method as Exponential Wavelet Iterative Shrinkage/Threshold Algorithm (EWISTA), and took advantage of the sparser representation of EWT, the simplicity of FISTA, and the fine-tuning performance of SISTA, with the aim of improving the reconstruction quality and accelerating the computation.

The rest of the paper is structured as follows. In Section 2, we review recent works related to sparse representation and reconstruction algorithms. Section 3 describes the methodology of the proposed EWISTA method with mathematical convergence analysis and we defined the evaluation measures. In Section 4, we designed the experiments to validate and compare our method with existing ones. Section 5 discusses the observations from the experiments, which confirmed the effectiveness and rapidness of the proposed method. Section 6 is devoted to the conclusion of the research conducted and future possibilities for further research. For the convenience of readers, the acronyms and their definitions are listed in Table 14 in the appendix.

2. Literature review

This section presents the related work of dealing with sparse representation and CS-MRI reconstruction. We searched the relevant keywords in "Web of Science Core collection", selected high-citation papers, and tried to give a relatively detailed review.

2.1. Sparse representation

Discrete WT (DWT) is the most common sparsifying transform, and is widely applied in a variety of academic and industrial fields [52,6,11]. Based on DWT, scholars have proposed more efficient variants than simple WT. Plonka [38] introduced easy path wavelet transform (EPWT), which takes pathways along through the array of function values and exploits the local correlations of the data in a simple and appropriate manner. The usual discrete orthogonal and biorthogonal wavelet transform can be formulated in EPWT. Khalidov et al. [21] proposed a new framework to extract the activity-related component, and termed the new wavelet basis as "activelets". Selesnick [40] studied the tunable Q-factor wavelet transform (TQWT), and found it was well suited for fast algorithms for sparsity-based inverse problems, because it is a Parseval frame, easily invertible, and can be efficiently implemented using radix-2 FFTs. Hao et al. [15] suggested to use contourlet as a new sparse transform. Ning et al. [31] suggested to use patch-based directional wavelets (PBDW) that trained geometric directions from undersampled data. PBDW had better performance in preserving image edges than conventional sparsifying transforms. Tang et al. [41] proposed a two manifold-based sparse representation that exploited the local structure of the test samples in corresponding sparse representations for enforcing smoothness across neighboring samples' sparse representation. Huang et al. [17] developed a Bayesian nonparametric model for reconstructing MRIs from highly undersampled k-space data. They performed dictionary learning as part of the image reconstruction process. Qu et al. [39] designed a patch-based nonlocal operator (PANO) to sparsify MR images by making use of the similarity of image patches. Kayvanrad et al. [20] showed that the visual pseudo-Gibbs artifacts can be greatly reduced by penalizing the translation-invariant stationary wavelet transform (SWT) coefficients; hence, they applied SWT for under-sampled MRI reconstruction. Paquette et al. [32] compared different sampling strategies and sparsifying transforms. They found that the DWT with Cohen–Daubechies–Feauveau 9/7 wavelets and uniform angular sampling in combination with random radial sampling showed better than other tested techniques to accurately reconstruct the ensemble average propagator (EAP) and its features. Pejoski et al. [37] used the discrete non-separable shearlet transform (DNST) as a sparsifying transform and the FISTA for reconstruction. Fang et al. [10] took signals as a sparse linear combination under an unknown transform order in fractional Fourier transform (FRFT) domain.

Nevertheless, these algorithms were reported to hinder computation resources. Our past work showed that EWT simply calculates the exponent of WT and can both increase the sparsity and reduce computation time [53]. Therefore, EWT was chosen as the sparse representation of this study.