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Group aggregation of pairwise comparisons using multi-objective optimization

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A B S T R A C T

In group decision making, multiple decision makers (DMs) aim to reach a consensus ranking of alternatives in a decision problem. The differing expertise, experience and, potentially conflicting, interests of the DMs will result in the need for some form of conciliation to achieve consensus. Pairwise comparisons are commonly used to elicit values of preference of a DM. The aggregation of the preferences of multiple DMs must additionally consider potential conflict between DMs and how these conflicts may result in a need for compromise to reach group consensus.

We present an approach to aggregating the preferences of multiple DMs, utilizing multi-objective optimization, to derive and highlight underlying conflict between the DMs when seeking to achieve consensus. Extracting knowledge of conflict facilitates both traceability and transparency of the trade-offs involved when reaching a group consensus.

Further, the approach incorporates inconsistency reduction during the aggregation process to seek to diminish adverse effects upon decision outcomes. The approach can determine a single final solution based on either global compromise information or through utilizing weights of importance of the DMs.

Within multi-criteria decision making, we present a case study within the Analytical Hierarchy Process from which we derive a richer final ranking of the decision alternatives.

1. Introduction

Multi-Criteria Decision Making (MCDM) seeks to determine the suitability of alternative outcomes of a decision with respect to several criteria. The concept of Pairwise Comparison (PC) is employed by many MCDM methods [29,30,16,4,23]. PCs enable the breaking down of a larger decision problem into more manageable smaller chunks helping to facilitate a separation of concerns. Each PC allows a Decision Maker (DM) to consider only a pair of decision elements and to determine their preference with respect to an intangible factor. From a set of PCs, one for each pairing of elements in a set of decision elements, a one-dimensional weights vector can be derived representing a ranking of the set of elements under consideration by the DM. PCs can be utilized both within a single decision making environment and within a group decision making environment.

For many real world decisions the opinion of multiple DMs is utilized, either to avail of their combined expertise or to incorporate conflicting views and experiences. In both cases there may be a level of disagreement and variance between

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the DMs, making the process of synthezising the DMs' judgments important. When utilized within a group environment, the process of deriving a weights vector from multiple DMs judgments needs to incorporate the aggregation of the group of DMs' judgments into the formulation of a single weights vector for the group.

Additionally, DMs are subject to irrationalities and various inconsistencies. Although inconsistencies can be used as a criterion to measure variability or diversity in the DMs' individual solutions and seek 'novelty' in the solution space, our approach is concerned with finding an aggregated group solution. From this perspective the inconsistencies can adversely affect the group decision, and aggregated solutions derived from inconsistent judgment data are not as valuable as solutions from potentially more consistent data. Therefore, seeking to reduce inconsistency during the aggregation process can help reduce its effects on the group decision. In the context of PC data, the inconsistency is the extent to which the set of the PC judgments are incoherent and when present can affect the accuracy of any derived weights vector [18].

This paper presents an approach to the aggregation of PC judgments of a group of DMs, whilst simultaneously seeking to reduce inconsistency. The approach utilizes Multi-Objective Optimization (MOO) to model compromise between each DM with respect to their judgments and an aggregation solution. MOO seeks to optimize multiple objectives simultaneously to find a set of trade-off solutions between the conflicting views of each DM. For these solutions, improvement in any one objective of the problem will result in a decrease within one or more of the other objectives. Together they map out the trade-off front of the problem. Analysis and exploration of the set of trade-off solutions facilitates better understanding and visibility of the underlying conflict within the group and of the compromise required in the reaching of consensus. Such analysis enables a more traceable and transparent approach to group decision making, one that identifies the trade-offs involved when looking to reach a consensus between a group of DMs.

A range of measures have been defined that can be utilized as objectives to measure the compromise between a DM's judgments and an aggregated solution. From an aggregated solution a single weights vector for the group can then be derived. Constraints can be utilized within the approach by each DM, to represent their tolerance of compromise regarding the amount of concession they will tolerate in the pursuit of finding aggregated solutions. By modeling the trade-off front between the objectives, both overall understanding of the nature of the problem and knowledge regarding the conflict between the DMs can be gleaned. Obtaining knowledge about the nature of the trade-off front should enable the setting of more feasible constraints by the DMs.

The approach can then select a single solution from the set of trade-off solutions based on utilizing knowledge of the global level of compromise of the group to reach consensus. Alternatively the approach can accommodate weights of importance, representing the significance of each DM, which can be used to find a weighted solution from the trade-off front in proportion to these weights. Further, the locating of the trade-off front allows sensitivity analysis of the effects of altering the DM weights to be swiftly carried out without needing to re-run the overall approach.

The rest of the paper is structured as follows: Section 2 defines and discusses the problem of group aggregation of PCs, and approaches to judgment syntheses and inconsistency reduction; our approach to group aggregation of PCs is then outlined and defined in Section 3; examples of the approach are discussed in Section 4 and a case study of an Analytical Hierarchy Process (AHP) [29] MCDM problem is presented in Section 5; finally conclusions are discussed in Section 6.

2. Aggregation of PC judgments of a group of DMs

The proposed approach deals with the aggregation of PC judgments of a group of DMs. PC allows a DM to consider only a pair of decision elements and to determine their preference, and strength of preference, between the pair, with respect to an intangible factor. This segmentation of a larger decision problem can be achieved through the use of the Law of Comparative Judgment [32]. Given two elements x and y, we denote that the DM prefers element x to element y with the notation $x \succ y$. Various numerical scales may be utilized to represent the strength of preference; the most widely utilized being the Saaty 1–9 scale [27], where, for example, if element x is preferred 3 times more than element y, this can be denoted as $x \succ y$ with a preference strength of 3. If neither element is preferred over the other, the elements are said to be equally preferred, denoted by $x \sim y$ and represented with the value 1.

Various other scales have been proposed of differing preference strength intervals, such as the Power scale [14], the Geometric scale [20], and the Logarithmic scale [17]. The examples presented of our approach utilize the 1–9 scale; however any bounded numerical scale can be utilized within the approach.

The set of PCs, one for each pairing of elements in a set of elements, along with the self-comparison values and the reciprocal values, can be collated into a two-dimensional Pairwise Comparison Matrix (PCM), as shown in (1) for a set of n elements, where a_{ij} represents the judgment between elements $i$and j.

$$PCM = \begin{pmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 1/a_{12} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \\ 1/a_{1n} & 1/a_{2n} & & 1 \end{pmatrix} \tag{1}$$