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Entailment and symmetry in confirmation measures of interestingness

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ABSTRACT

In a recent paper Greco et al. (2012) propose a number of properties for measures of rule interestingness. The most fundamental of these properties is that such measures should be Bayesian confirmation measures and this criterion provides the context for the current paper as well. They also propose a number of properties relating to entailment and symmetry in order to discriminate between various confirmation measures which have been proposed in the literature. Working within the same framework of confirmation measures, several limitations of their proposed properties are discussed and a motivation provided for alternative properties. Two new measures of interestingness are proposed and then compared with two other recently proposed measures which also satisfy these properties.

1. Introduction

An important issue in the field of knowledge discovery is the ability to quantify discovered rules in terms of how interesting they are. One feature of research on this topic is that there are numerous non-equivalent measures of rule 'interestingness' (for reviews see [5,11]). In order to discriminate between these measures many properties have been discussed and compared in the literature, but there does not seem to be a general consensus on which properties an adequate measure should satisfy (see for example [10,13–15]).

A significant contribution to research on this topic has been made by Greco et al. [8] who argue that measures of interestingness should be Bayesian confirmation measures. This narrows the field of available measures and by drawing on work in the philosophy of science, particularly Crupi et al. [2], they narrow the field further by identifying entailment and symmetry properties that measures of interestingness should satisfy. They also provide new approaches to normalization of confirmation measures in order to obtain new measures that satisfy the desired properties.

In the context of entailment, Greco et al. review the properties Ex_1 and logicality, which have been discussed in the philosophical literature. Ex_1 requires that the degree of confirmation of hypothesis H by evidence E should be greater if E entails H than if it does not and smaller if E refutes H, i.e. E entails the negation of H, than if it does not. Logicality states that the confirmation of H by E should be maximal if E entails H and minimal if E refutes H. Greco et al. propose their own version of logicality, called weak L, according to which confirmation is maximal if E entails H and $\neg E$ entails $\neg H$ and minimal if E entails $\neg H$ and $\neg E$ entails H. In a similar way, they propose their own version of Ex_1, called weak Ex_1, which will be explored in Section 3.

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In the context of symmetry, Greco et al. consider symmetry properties proposed by Crupi et al. These are an extension of an earlier set of symmetry principles proposed by Eells and Fitelson [3] and are based on the idea that confirmation should be a generalization of classical logic. The idea is that symmetries in confirmation should reflect symmetries in entailment/refutation relations in classical logic. These symmetry properties will be explored in Section 4.

The approach adopted here has much in common with that of Greco et al. and so should be seen as a refinement rather than rejection of their work. There is agreement that the properties Ex_1 and logicality are inappropriate for measures of rule interestingness and the reasons for reaching this conclusion are similar. However, it is argued that the recently proposed property of maximality/minimality [6] captures their insights more accurately. In terms of symmetry, it is argued that while the properties of Crupi et al. may be appropriate for their conception of confirmation as a generalization of classical logic, they are inappropriate for measures of rule interestingness. Instead, an alternative is provided which closely corresponds to the proposal of Eells and Fitelson [3].

Two new Bayesian confirmation measures of rule interestingness are also proposed to satisfy these properties and it is noted that two recently proposed measures also satisfy them [6]. These measures are discussed and compared with those proposed by Greco et al.

The structure of the rest of the paper is as follows. After presenting some formal notation in Section 2, the entailment properties proposed by Greco et al. and alternatives to them are discussed in Section 3 and the same is done for symmetry properties in Section 4. The new confirmation measures are proposed in Section 5 and compared with existing measures. Finally, conclusions are presented in Section 6.

2. Preliminaries

Following Greco et al. a rule induced from a dataset on a universe U will be denoted $E \rightarrow H$ where E represents the premise (or evidence) and H the conclusion (or hypothesis). Support, denoted $sup(\cdot)$, represents the number of objects in the dataset satisfying a given attribute. For example, $sup(E)$ is the number of objects satisfying the premise of the rule. The following notation will be used: $a = sup(H,E),\ b = sup(H,\neg E),\ c = sup(\neg H,E)$ and $d = sup(\neg H,\neg E)$. $a,\ b,\ c$ and d can be regarded as frequencies that can be used to estimate probabilities, e.g. $Pr(E) = (a+c)/|U|$ or $Pr(H,E) = a/|U|$.

An interestingness measure $c(H,E)$ has the property of Bayesian confirmation if and only if it satisfies the following conditions:

$$c(H,E)\begin{cases} >0 & \text{if } Pr(H|E) > Pr(H) \\ =0 & \text{if } Pr(H|E) = Pr(H), \\ <0 & \text{if } Pr(H|E) < Pr(H) \end{cases} \tag{1}$$

E is said to confirm H if and only if $c(H,E) > 0$ and to disconfirm H if and only if $c(H,E) < 0$. All of the interestingness measures considered by Greco et al. satisfy this property and so it will be assumed in this paper as well. Such measures will be referred to as confirmation measures. In order to discriminate between confirmation measures other properties have been proposed as well. Some of these will now be considered.[1]

3. Entailment

In accordance with their view of confirmation as an extension of classical logic Crupi et al. [2] proposed a property Ex_1 for confirmation measures. They first consider a function v such that $v(H,E)$ has the same positive value (e.g. $+1$) if E entails H, $v(H,E)$ has the same negative value (e.g. -1) if E refutes H and $v(H,E) = 0$ otherwise. For a confirmation measure $c(H,E)$, they then define Ex_1:

$$\mathbf{Ex_1}:\ \text{If } v(H_1,E_1) > v(H_2,E_2) \text{ then } c(H_1,E_1) > c(H_2,E_2).$$

Ex_1 guarantees that $c(H,E)$ will be greater in cases where E entails H than in other cases and $c(H,E)$ will be less in cases where E entails $\neg H$ than in other cases. In a probabilistic context, it will be assumed that $Pr(H|E) = 1$ is equivalent to E entails H. Hence, when considering interestingness measures of rules, Ex_1 requires that they will be greater when there are no counterexamples (i.e. when $c = sup(\neg H,E) = 0$) than in other cases and less when there are no instances of the rule (i.e. when $a = sup(H,E) = 0$) than in other cases.

Greco et al. point out a problem with Ex_1 in the case of confirmation by asking us to consider the following cases:

Case 1: $a = 100,\ b = 99,\ c = 0,\ d = 1.$
Case 2: $a = 99,\ b = 0,\ c = 1,\ d = 100.$

They point out that the degree of confirmation should be greater in case 1 than in case 2 according to Ex_1. Yet they note that the difference between $Pr(H|E) = 1$ and $Pr(H|\neg E) = 0.99$ is very small in case 1 and much larger in case 2 (since

[1] Throughout this paper, it is assumed that E and H are contingent and that $0 < Pr(E) < 1$ and $0 < Pr(H) < 1$. In terms of $a,\ b,\ c$ and d, this means that cases where $a = b = 0$ or $c = d = 0$ or $a = c = 0$ or $b = d = 0$ are not considered.

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