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An optimal and unifying vector fitting method for frequency-domain system identification

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ABSTRACT

This paper proposes an optimal and unifying instrumental variable (IV)-based Vector Fitting (VF) method for frequency-domain (FD) identification of models formed by rational basis function (RBF) expansions. The proposed method is denoted by IV-FD-VF and can be similarly applied for estimating models formed by both continuous- and discrete-time RBF sets. The key advantage of IV-FD-VF lies in the fact that, differently from standard FD-VF approaches, it guarantees an optimal solution after convergence. This important optimality property is proved to be independent of the nature of the noise that corrupts the data (for instance, if it is white or coloured). Two case studies are used to validate the proposed IV-FD-VF method. One of these case studies considers actual frequency-domain data sets extracted from two different single-phase power transformers.

1. Introduction

System identification based on linear models formed by rational basis functions (RBFs) plays a key role in various areas of engineering [1,2]. In power systems, for instance, the so-called *vector fitting* (VF) algorithms have become extremely popular due to their capability of estimating RBF models that meet real-world physical requirements such as realness, causality, reciprocity (in the multiport case), stability and passivity [2]. Most successful applications of VF in power systems comprehend wideband frequency response modelling of transmission lines and transformers [3–7], transient analysis of frequency-dependent network equivalents [8–11] and estimation of oscillatory (electromechanical) oscillations through ringdown analysis [12,13]. In the particular case of frequency response modelling of power transformers, estimating wideband dynamic models may improve electromagnetic transient simulations which subsidize, for instance, contingency analysis and equipment insulation design [14,15] (see also [16] for a connection between very fast transients and paper insulation in power transformers). Microwave and vibration analyses can also be considered as application areas for VF algorithms [17,18].

VF implementations, which are also understood as robust reformulations of the original Sanathanan-Koerner [19] and Steiglitz-McBride [20] iterations [21], essentially estimate RBF model parameters by transforming the minimization of a nonlinear least-squares objective function (NLSOF) into a sequence of linear least-squares problems [2]. In other words, the original nonlinear optimization problem is rewritten as an alternative iterative procedure composed by a sequence of linear optimization problems.

By making use of frequency-domain (FD) tabulated data, the VF method has been firstly proposed by Gustavsen and Semlyen [22] for estimating models given by rational transfer functions. Particularly, such a FD VF (FD-VF) method uses continuous-time partial fraction functions as RBF sets for representing these models. Few years later, several modifications and improvements have been incorporated within the original VF procedure described in [22]. By promoting some numerical enhancements for faster convergence (the QR approach) [23,24] and also the so-called VF relaxation [25], for instance, FD-VF have reached the so-called *vectfit3* form, found in [26], which is still one of the most popular implementations of FD-VF.

In [27], the authors introduced another standard FD-VF algorithm, known as Orthonormal VF, which consists of replacing the original partial fractions with the so-called continuous-time Takenaka-Malmquist orthonormal basis functions [28]. Improvement in terms of numerical conditioning is considered as one of the benefits of using orthonormal basis functions as RBFs [27,29]. In [30], the use of frequency localizing basis functions as RBFs is also studied in the context of FD-VF. Discrete-time counterparts of these FD-VF algorithms can be found, for instance, in [29,31,32].

Nonetheless, although standard VF algorithms are worldwide recognized for providing fast and good estimates for FD system identification problems (and that is actually the main reason for the success of these algorithms), two key issues related to them still remain. First,

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there is no proof of convergence for the VF iterations. Second, they do not guarantee their converged solutions are local (or the global) optimums of their corresponding NLSOFs [2]. In fact, if data used for estimation are corrupted by coloured noise, some recent results on VF implementations show they never converge to any local minimum [33].

Now, as far as system identification based on time-domain (TD) tabulated data is concerned, TD VF methods present a similar characteristic, that is, they also do not guarantee their converged solutions are local (or the global) optimums of their corresponding NLSOFs. However, in this particular context of TD VF, [11] recently proposed an instrumental variable (IV) discrete TD VF (IV-dTD-VF) method which effectively overcomes such an issue. In fact, assuming convergence of the VF iterations, IV-dTD-VF guarantees that the solution is necessarily a local optimum of its corresponding NLSOF.

The objective of this paper is to propose an extension of the IV-dTD-VF method proposed in [11] for frequency-domain system identification. The terminology 'unifying' is also used in the method to emphasize it can be similarly applied for estimating models formed by both continuous- and discrete-time RBF sets. We denote such a unifying frequency-domain approach by IV-FD-VF. The key advantage of IV-FD-VF lies in the fact that, differently from standard FD-VF implementations, it guarantees that the gradient local optimality condition of its NLSOF is necessarily satisfied after convergence. Moreover, this important result does not depend on the nature of the noise that corrupts the data. As a consequence, more accurate RBF models may be obtained even if estimation data are corrupted by coloured noise.

The paper is organized as follows. In Section 2, we formulate the identification problem of estimating linear RBF models in the frequency-domain. We also establish in this section a unifying FD-VF method. In Section 3, we transform the unifying FD-VF method of Section 2 into the proposed IV-FD-VF iterations. In Section 4, two case studies are used to validate the proposal. The first case study deals with a continuous-time example where actual frequency-domain data samples were extracted from two different power transformers, whereas the second case study aims at identifying a third order discrete-time system corrupted by coloured noise. Finally, Section 5 addresses the conclusions of this work.

2. Problem Statement and a unifying FD-VF method

A stable single-input single-output (SISO) linear time-invariant system can be described in terms of its scalar input $U_0(\alpha)$ and its scalar output $Y_0(\alpha)$ as

$$Y_0(\alpha) = G_0(\alpha)U_0(\alpha) + V(\alpha), \qquad \alpha = z \text{ or } s \tag{1}$$

where $V(\alpha)$ represents the additive disturbance at the system output, and α determines if the system is described either in continuous-time ($\alpha = s$) or discrete-time ($\alpha = z$).

In this paper, we deal with the frequency-domain system identification problem of estimating a RBF model for $G_0(\alpha)$ based on a set of noisy frequency response data samples $\{G_0'(\alpha_k), \alpha_k\}$, $k = 1, \cdots, N$, where each term α_k is associated with a frequency ω_k according to one of the following relations:

$$\alpha_k = \begin{cases} s_k = j\omega_k & \text{(continuous-time case)} \\ z_k = e^{j2\pi\omega_k/\omega_s} & \text{(discrete-time case)} \end{cases} \tag{2}$$

where ω_s denotes sampling frequency. Note that an additive noise component $V_U(\alpha_k)$ appears in the measured frequency response of the system $G_0'(\alpha_k)$ due to the input-output relation in (1):

$$\frac{Y_0(\alpha_k)}{U_0(\alpha_k)} = G_0(\alpha_k) + V_U(\alpha_k) = G_0'(\alpha_k), \tag{3}$$

where $V_U(\alpha_k) = V(\alpha_k)/U_0(\alpha_k)$.

The desired RBF model must have a n-th order transfer function structure in the form

$$G(\alpha) = c_0 + \sum_{i=1}^{n} c_i\Phi_i(\alpha, \mathbf{a}), \tag{4}$$

where $\{c_i\}$ are the unknown model structure coefficients and $\{\Phi_i(\alpha, \mathbf{a})\}$ denotes a set of n rational basis functions which are completely parametrized by the unknown poles of $G(\alpha)$, here grouped into vector $\mathbf{a} = [a_1 \ \cdots \ a_n]^T$(with $(\cdot)^T$ denoting the transpose operator).

In this paper, it is also assumed that model structure (4) can be represented by a linear state-space realization given by

$$\begin{cases} \alpha\mathbf{X}(\alpha) &= \mathbf{A}\mathbf{X}(\alpha) + \mathbf{B}U_0(\alpha), \\ Y(\alpha) &= [c_1 \ \cdots \ c_n]\mathbf{X}(\alpha) + c_0U_0(\alpha), \end{cases} \tag{5}$$

which means that the pair of matrices $\mathbf{A} \in \mathbb{C}^{n\times n}$ and $\mathbf{B} \in \mathbb{C}^{n\times 1}$ must necessarily satisfy the well known state condition

$$\mathbf{\Phi}(\alpha) = \frac{\mathbf{X}(\alpha)}{U_0(\alpha)} = [\Phi_1(\alpha, \mathbf{a}) \ \cdots \ \Phi_n(\alpha, \mathbf{a})]^T = (\alpha\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}, \tag{6}$$

with $\mathbf{I}$ being the $n \times n$ identity matrix. From Eq. (6), it is clear that matrices $\mathbf{A}$ and $\mathbf{B}$ define the RBF set used in the model structure. In principle, any continuous- or discrete-time set $\{\Phi_i(\alpha, \mathbf{a})\}$ that fits into definition (6) can be used as RBFs.

When it comes to continuous-time system identification, perhaps the most common choice is to use a set of partial fractions as RBFs [2,34], i.e.,

$$\Phi_i(s, \mathbf{a}) = \frac{1}{s-a_i}, \quad i = 1, \cdots, n. \tag{7}$$

For this particular case, $\mathbf{A}$ and $\mathbf{B}$ are as follows

$$\mathbf{A} = \begin{bmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_n \end{bmatrix}; \qquad \mathbf{B} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}. \tag{8}$$

Meanwhile, discrete-time approaches many times consider models formed by the so-called discrete-time Takenaka-Malmquist orthonormal basis functions [29,1]

$$\Phi_i(z, \mathbf{a}) = \frac{\sqrt{1-|a_i|^2}}{z-a_i} \prod_{j=1}^{i-1} \left(\frac{1-a_j^*z}{z-a_j}\right), \quad i = 1, \cdots, n \tag{9}$$

where $(\cdot)^*$ denotes the conjugate operator. The corresponding state-space construction for the RBF set in (9) can be found, for instance, in [28]. In fact, in [28] it is listed the corresponding state-space construction of several continuous- and discrete-time RBF sets for which (6) holds.

Estimating a RBF model in terms of its poles $\mathbf{a}$ and coefficients $\{c_i\}$ requires the definition of a certain estimation criterion. By means of the absolute weighted least-squares criterion, estimating $\mathbf{a}$ and $\{c_i\}$ becomes the following nonlinear optimization problem:

$$\begin{aligned} &\underset{\mathbf{c},\mathbf{a}}{\operatorname{argmin}} \sum_{k=1}^{N} |W(\alpha_k)(G_0'(\alpha_k)-G(\alpha_k))|^2, \\ &= \underset{\mathbf{c},\mathbf{a}}{\operatorname{argmin}} \sum_{k=1}^{N} \left| W(\alpha_k)\left(G_0'(\alpha_k)-\left(c_0 + \sum_{i=1}^{n} c_i\Phi_i(\alpha_k, \mathbf{a})\right)\right)\right|^2, \end{aligned} \tag{10}$$

where $\mathbf{c} = [c_0 \ \cdots \ c_n]^T$ and $W(\alpha_k)$ is a weighting function to be selected by the user. Choosing the weighting function $W(\alpha_k)$ goes beyond the scope of this paper, although the interested reader is hereby referred to reference [35], which analyses the adoption of different weighting functions in frequency-domain system identification.

The main objective of this paper is to propose the IV-FD-VF iterations, which consist of an optimal way for estimating the RBF model parameters $\mathbf{c}$ and $\mathbf{a}$. In fact, IV-FD-VF may be considered as a unifying instrumental variable version of the standard FD-VF iterations, since it is also based on transforming (10) into a sequence of linear problems where coefficient sets are estimated by means of pre-specified update-dependent poles.